Preface

For global fields of positive characteristic p, Drinfeld and later Anderson defined objects, now called Drinfeld modules and A-motives, which bear many similarities to abelian varieties over number fields. The Tate modules of these objects provide interesting examples of strictly compatible systems of Galois representations. In many cases they also possess an analytic uniformization and a kind of Hodge theory, which altogether can be viewed as étale, Betti and de Rham realizations.

A natural next step was taken by Goss, who assigned an L-function to any family of such objects over a variety of finite type over $\mathbb{F}_p$, which is a power series in a variable T. By analogy to the L-function of a constructible ℓ-adic sheaf in the case $\ell \neq p$, Goss conjectured that this L-function is, in fact, a rational function of T. In 1996 Taguchi and Wan proved this conjecture, using analytic methods à la Dwork. This development parallels that around the Weil conjecture, where Dwork's analytic proof of rationality predated the algebraic proof of Grothendieck et al. based on ℓ-adic étale cohomology.

The present monograph performs some of the next logical steps in this direction. It provides a set of algebro-geometric and homological tools to give a purely algebraic proof of Goss's rationality conjecture and – hopefully – much more. Specifically, we introduce a notion of *crystals*, which contains families of A-motives as special cases, and develop a complete cohomological theory for them. We also prove a Lefschetz trace formula that expresses the L-function in terms of explicitly computable cohomology groups. These cohomology groups contain finer information than the analytic rationality proof and will deserve further study. We also describe their relation to the cohomology of étale sheaves of $\mathbb{F}_p$-modules over varieties in characteristic p.

The book is intended for researchers and advanced graduate students interested in the arithmetic of function fields and/or cohomology theories for varieties in positive characteristic. A prerequisite to reading this book is a good working knowledge in algebraic geometry as well as familiarity with homological algebra and derived categories. Most of the necessary background can be found, for instance, in the books [26] on *Algebraic Geometry* by Hartshorne and [47] on *Homological Algebra* by Weibel. From that point on, we have tried to be largely self-contained.

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