Available online at www.sciencedirect.com
ScienceDirect
Historia Mathematica 38 (2011) 123–135
HISTORIA MATHEMATICA
www.elsevier.com/locate/yhmat

Book Reviews

Thomas Harriot's Doctrine of Triangular Numbers: The 'Magisteria Magna'
Edited by Janet Beery and Jacqueline Stedall. Zürich (European Mathematical Society). 2008. ISBN 978-3-03719-059-3. 144 pages. €64.00.

Thomas Harriot (1560–1621) was a brilliant character in the realm of Elizabethan scientific thought. After graduating from Oxford University, he was sent by Sir Walter Raleigh on the Grenville expedition to America. Drawing on his experiences, Harriot wrote *A briefe and true report of the new found land of Virginia* [Harriot, 1951]. The book was published in 1588 and was to be his only publication during his lifetime. It includes an account of the native population including their customs, much diverse information on the flora and fauna of the region, and the items found there that were potential commodities, especially tobacco. The 1590 edition includes John White's drawings of plants, animals, and indigenous people [Harriot, 2007]. Among his other contributions to science, Harriot observed and recorded Halley's Comet in 1607, his observations enabling Wilhelm Bessel to calculate its orbit in 1804. Harriot was also the first to ascertain the law of refraction later discovered by the Dutch scientist Willebrord Snell. After learning about the creation of a primitive refracting telescope by Hans Lippershey in Holland, Harriot constructed his own instrument with magnification power of six. He pointed it toward the heavens at roughly the same time as Galileo in Italy, Nicolas Peirese in France and Christoph Scheiner in Bavaria. All, including Harriot, now have craters on the moon named in their honor. Harriot saw sunspots, the moons of Jupiter, and made the first telescopic drawings of the moon. However, unlike Galileo, he failed to publish his observations.

Harriot was well-read mathematically and an algebraic innovator. Indeed, he was arguably England's greatest mathematical scientist before Sir Isaac Newton. His *Artis analyticae praxis*, edited by William Warner and published posthumously in 1631, contains his work on the theory of equations and the solution of polynomial equations [Seltman and Goulding, 2007]. In this, Harriot was greatly influenced by the mathematics of François Viète, and many pages of his manuscripts contain worked out problems from Viète's writings. In turn, Harriot's work influenced that of John Wallis, whose *Arithmetica infinitorum* later had a great impact on Newton.

The origin of triangular numbers can be traced back to the Greeks. They were highlighted in the second-century philosophical manuscripts *On mathematical matters useful for reading Plato* by Theon of Smyrna and the *Introduction to arithmetic* by Nicomachus of Gerasa. The latter was translated into Latin by the fourth-century philosopher Iamblichus of Chalcis and in the sixth century by the classicist Boethius. It was through Boethius' *Introductio* that Thomas Harriot learned of triangular numbers as found in the third row and column of the following array:

1	1	1	1	1	1	...
1	2	3	4	5	6	...
1	3	6	10	15	21	...
1	4	10	20	35	56	...
1	5	15	35	70	126	...
1	6	21	56	126	252	...
...	...	...	...	...	...	...

By 1618, Harriot had completed a 38-page manuscript entitled, *Doctrine of Triangular Numbers: the 'Magisteria Magna'*. The manuscript appears here in facsimile with a 50-page introductory essay by the editors on the historiography of the manuscript. The method, worked out in considerable generality and employed by Harriot, is important because the relevant formulas are displayed in something very close to modern algebraic notation using letters and operational signs as opposed to the wordy descriptions used by his predecessors. The one major algebraic exception was his failure to use exponentiation. The mathematics that appears in the text is of significant interest, for Harriot was the first to realize and work out the details showing that any sequence of numbers with constant nth differences can be generated by a polynomial function and conversely that polynomial equations evaluated at regular intervals produce constant differences. He worked with numerical examples and also took a more algebraic approach. For example, expressing an algebraic finite difference array in reverse, as shown below,

			d
		c	
	b		$d + c$
a		$c + b$	
	$b + a$		$d + 2c + b$
a		$c + 2b + a$	
	$b + 2a$		$d + 3c + 3b + a$
a		$c + 3b + 3a$	
	$b + 3a$		$d + 4c + 6b + 4a$
a		$c + 4b + 6a$	

he recognized the appearance of the generalized triangular numbers or binomial coefficients. He used that fact to determine a generating polynomial for difference sequences which eventually have a common difference. For example, if the sequence was given algebraically by d, $d + c$, $d + 2c + b$, $d + 3c + 3b + a$, $d + 4c + 6b + 4a$, ... as in the fourth column, the third differences were constant as shown above. By working backwards, Harriot concluded that the algebraic sequence is generated by the third-degree polynomial:

$$d + c \cdot (n-1) + b \cdot \frac{(n-1)(n-2)}{1 \cdot 2} + a \cdot \frac{(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3},$$

where a, b, c, and d are the elements on the upper diagonal of the array. He realized that by working backwards, the next term of the sequence could be computed. He used the technique to generate the general term for numerical sequences, to generate general formulas for polyhedral numbers, and to calculate polynomial expressions for a cube plus three times its root, a cube plus a square plus its root, and cubes and fourth powers. He discovered that the sequence consisting of every kth term led to constant differences as did the kth term of the mth differences. He constructed many of these partial arrays and realized that it was possible to reconstruct the original array from a partial array by interpolating new values between the existing values in a consistent way by means of constant differences. Harriot's method of interpolating logarithms was similar but not identical to the method of his contemporary Henry Briggs, and his interpolation formula was rediscovered by Newton and James Gregory in the 1660s.

It is remarkable that it has taken nearly 400 years for this manuscript to appear in print. In the book's introduction, there is a discussion on how Harriot's new mathematical ideas were disseminated among a small group of his friends—none of whom found the time to write a commentary explaining his symbolic interpolation formulas or were capable of understanding this new interpretation. Undoubtedly, part of the problem was the lack of any explanation by Harriot of his mathematical accomplishments. Further delaying its publication, the manuscript was lost for over a century before it turned up in 1784 at Petworth House in Sussex, England. All these matters are discussed in detail in this edition. There is also valuable commentary with each manuscript page. The editors, building on their extensive knowledge of Harriot and his mathematics (see, for example, Stedall [2003] and Beery [2004, 2007]), have done exemplary work researching the history and evolution of the manuscript and explaining the mathematics involved. For lovers of the history of 17th-century mathematics and algebra in particular, this book is a highly recommended addition to the mathematical literature.

References

Beery, J.L., 2004. Thomas Harriot's treatise on figurate numbers. In: Proceedings of the Canadian Society for History and Philosophy of Mathematics, 30th Annual Meeting, pp. 24–43.

Beery, J.L., 2007. Navigating between triangular numbers and trigonometric tables: how Thomas Harriot developed his interpolation formulas. In: Proceedings of the Canadian Society for History and Philosophy of Mathematics, 32nd Annual Meeting, pp. 37–41.

Harriot, T., 1951. A Briefe and True Report of the New Found Land of Virginia (1588). Clements Library Association, Ann Arbor.

Harriot, T., 2007. A Briefe and True Report of the New Found Land of Virginia (1590). University of Virginia Press, Charlottesville.

Seltman, M., Goulding, R., 2007. Thomas Harriot's Artis Analyticae Praxis: An English Translation with Commentary. Springer, New York.

Stedall, J.A., 2003. The Greate Invention of Algebra: Thomas Harriot's Treatise on Equations. Oxford University Press, Oxford.

James J. Tattersall
Department of Mathematics,
Providence College,
1 Cunningham Square,
Providence, RI 02918, USA
E-mail address: tat@providence.edu

Available online 25 June 2010

10.1016/j.hm.2010.05.006

Duel at Dawn: Heroes, Martyrs and the Rise of Modern Mathematics

By Amir Alexander. Cambridge, Mass. and London (Harvard University Press). 2010. ISBN: 978-0-674-04661-0. 307 pp. US$28.95.

Amir Alexander has a flair for proposing novel links of mathematics to the wider culture. His *Geometrical Landscapes* [Alexander, 2002] argued the influence of the great overseas