

BOOK REVIEWS  
BEFORE CALCULUS

Janet Beery and Jacqueline Stedall (eds), *Thomas Harriot's doctrine of triangular numbers: the 'Magisteria Magna'*. European Mathematical Society, Zurich, 2009. Pp. viii + 137, £55.00 (hardback). ISBN 978-3-03719-059-3.

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In the history of the mathematical sciences it is fascinating to see how new challenges bring about new solutions. In early modern times, mathematicians were increasingly confronted with functional relations between quantities. These relations, in which one quantity depends on another, occurred in various contexts such as navigation and cartography, theories of motion, and celestial mechanics. The tools available to deal with them, such as the theory of proportions or diagrammatic representations of change, mostly had their roots in antiquity and had been further developed during the Middle Ages. Some of these tools, for example algebraic notation, were in rapid development. In the second half of the seventeenth century these developments led to the multiple invention of the calculus, which was to change in a fundamental way the method of dealing with functional relations. In the early seventeenth-century manuscript edited in *Thomas Harriot's doctrine of triangular numbers*, the English mathematician and philosopher Thomas Harriot (1560–1621) develops an ingenious tool for dealing with functional relations that hitherto has been paid little attention in the history of mathematics: the use of

difference tables to interpolate between given values of a function. The book thus shows what was possible in dealing with functional relations before the advent of the calculus.

Harriot's scientific life was fruitful and productive. He was a scientific adviser in the service of Sir Walter Raleigh, and a pioneer of telescopic observation and of early modern algebra. He contributed to such diverse fields as linguistics, navigation, botany, astronomy, optics and mechanics, but published hardly any of his scientific findings. The treatise 'Magisteria Magna', in which he presents his method of interpolation, remained unpublished for almost 400 years; it has now been edited by Janet Beery and Jacqueline Stedall.

Harriot's exposition is almost non-verbal but follows a clear and elegant line of reasoning. He starts with a very basic table. The first row consists of ones. Below this is a row of sums of the ones from the beginning to the place above the respective entry. The resulting sequence is that of the natural numbers: 1, 2, 3, 4, . . . . The next row is again a row of sums of the entries of the previous row: 1, 3, 6,

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10, . . . . These are the triangular numbers in a narrower sense, known since antiquity: each entry represents the number of dots in an equilateral triangle evenly filled with dots. And Harriot continues to produce rows of sums of the entries of the previous row, the next row being 1, 4, 10, 20, . . . . The numbers in such a table may be referred to as triangular in a broader sense, and it is in reference to this that Harriot's treatise is headed a 'doctrine of triangular numbers'.

Harriot then passes on to tables with sequences whose first element may be any natural number. On reading these tables backwards, that is, from a sequence of sums to the sequence of its summands, the sum table becomes a difference table. Such difference tables can be produced starting from any table of functional values, and in particular also values of transcendental functions such as the sine, tangent and logarithmic functions. Although in general the continuous taking of differences of differences will not yield a sequence of constant differences, one can use such tables for approximate interpolation by stipulating a given sequence of differences to be constant.

Harriot then describes the entries of difference tables in terms of the first elements of each sequence, using his convenient algebraic notation. By considering partial tables whose starting sequence consists of only every  $n$ th element of the starting sequence of a given difference table, he is able to derive general formulae of how the first elements of the original table depend on those of the partial one. By constructing a difference table using these formulae he is able to infer interpolation formulae expressing new functional values in terms of given ones and their differences, differences of differences, and so on. These interpolation formulae he calls 'Magisteria'.

The edition offers a facsimile reproduction of 40 pages of the manuscript source, with an explanatory commentary on facing pages. No transcription is needed, because Harriot's writing is clearly readable in this treatise, and his algebraic notation is nearly modern. Nevertheless the editors transcribe selected formulae into modernized notation to facilitate reading. Latin prose, which is very rare, is transcribed and translated into English.

The edition also offers a 53-page essay that introduces the topic of triangular numbers, difference tables and interpolation and thus provides a helpful preparation for reading the manuscript itself. The essay further discusses in considerable detail the history of the reception and influence of Harriot's manuscript treatise. The authors analyse the work on difference tables and interpolation by Nathaniel Torporley, Henry Briggs, Walter Warner, Charles Cavendish, John Pell, John Collins, Nicolaus Mercator, Isaac Newton and James Gregory, thereby demonstrating that difference tables were a much-discussed theme of English seventeenth-century mathematics and uncovering a network of informal communication between mathematicians. In particular, the authors reveal a hitherto unrecognized relation between Mercator and Warner.

Historians of mathematics will welcome this book as providing new insights into the development of mathematics before the invention of the calculus and as a model for the publication of historical sources on mathematics. Dealing with a neat, easily comprehensible aspect of mathematics in a historically exciting period, the book has the potential to raise the interest of mathematicians in history and of historians in mathematics.

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