

Preface

These are the lecture notes for a graduate course which the author taught twice, first at the ETH, Zürich in Spring - Summer 2007 as a part of the “Nachdiplom Lectures” series and then at the University of Michigan, Ann Arbor, in the Fall of 2007 as a regular graduate course with exercises and grades. One can view these lectures as a substantially extended version of [Ba07] (the course of [Ba07] consisted of five lectures only).

The goal of the course is by starting from basic linear algebra, to walk the reader towards most recent advances in enumeration of integer points in polyhedra. As in [Ba07] and before that in [BP99], the structural theory is developed with an eye on algorithmic applications. Compared to [BP99] the present text is much more suitable for studying, as the author hopes. It contains figures, exercises, and the background material on lattices and polyhedra with more or less complete proofs. It also contains some new results, most notably recent remarkable “local” formulas by N. Berline and M. Vergne [BV07], and a new look at some old results, such as the role played by the classical construction of continued fractions or the “continuity” property of polynomials enumerating integer points in parametric polytopes as viewed through the prism of identities in the algebra of polyhedra. On the other hand, unlike in [BP99], there is no discussion in these notes of the relevant algebraic geometry of toric varieties.

Exercises constitute an essential part of these notes. Problems marked with \circ are relatively straightforward and solving them is important for understanding the material. Problems marked with $*$ are considered to be of the research level, they often come with a reference to the literature.

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