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★ **An invitation to quantum groups and duality.**

From Hopf algebras to multiplicative unitaries and beyond.

EMS Textbooks in Mathematics.

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This book provides a complete overview of available operator algebra approaches to quantum groups, ranging from Woronowicz' compact quantum groups over Baaj and Skandalis' multiplicative unitaries to locally compact quantum groups as developed by Kustermans and the reviewer. The book is very well organized and entirely self-contained, in the sense that all definitions and concepts are carefully introduced. There are not so many proofs, but systematic references to the literature are provided. This book will be very useful for people interested in the abstract axiomatics of quantum groups, but less useful for those who are looking for examples and applications of the theory.

The functional analytically oriented reader will find the first part of the book very useful, since it provides the necessary algebraic background: an introduction to Hopf algebras and to Van Daele's multiplier Hopf algebras (with or without integrals). On the other hand, the appendix explains several concepts from the theory of C^* -algebras and von Neumann algebras.

The second and main part of the book starts with an introduction to Woronowicz' compact quantum groups. These can be described in a C^* -algebraic language, but also as a special class of Hopf $*$ -algebras. Both approaches are clearly explained and make the transition from a purely algebraic to a functional analytic approach to quantum groups. The quantum group $SU_q(2)$ is discussed in detail, as well as the universal compact quantum groups $A_o(F)$ and $A_u(F)$.

The concept of a multiplicative unitary is introduced and motivated from an algebraic point of view. The author singles out the family of what he calls well-behaved multiplicative unitaries: examples include the regular multiplicative unitaries of Baaj and Skandalis and the manageable/modular multiplicative unitaries of Woronowicz.

Part II ends with an overview of the theory of locally compact quantum groups due to Kustermans and the reviewer. Both the von Neumann algebraic and the C^* -algebraic definition are presented. The Haar measure gives rise to a multiplicative unitary, which then allows the author to construct the antipode and the dual locally compact quantum group. Two examples, both due to Woronowicz, are treated in detail: the quantum $E(2)$ and the quantum $az + b$ groups.

The last part of the book, entitled "Selected topics", has three chapters. The first one deals with Baaj and Skandalis' theory of crossed products of quantum group actions on C^* -algebras and the Takasaki-Takai biduality theorem. The remaining chapters concentrate on the author's own research, dealing with pseudo-multiplicative unitaries. These relate to multiplicative unitaries in the same way as groupoids relate to groups. There is not yet a satisfying theory of locally compact quantum groupoids, but the author's work explained in Chapters 10 and 11, together with several articles by M. Enock [see *J. Funct. Anal.* **223** (2005), no. 2, 311–364; [MR2142344](#); and references therein], goes in that direction.

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