# Preface

In this monograph we study the initial value problem (Cauchy problem) and the Dirichlet problem for a class of degenerate diffusions modeled on the equation $u_t = \Delta u^m$, $m > 0$, $u \geq 0$. Our approach to these problems is through the use of local regularity estimates and Harnack type inequalities, which yield equicontinuity and hence compactness for families of solutions. The theory is quite complete in the slow diffusion case (porous medium equation) $m > 1$ and in the super-critical fast diffusion case $m_c < m < 1$, where $m_c = (n-2)_+/n$, while problems remain open in the range $m \leq m_c$. In this book we have emphasized the techniques used in the proofs of the results presented, in the hope that they will have a wider scope of applicability beyond the specific problems discussed here. We have also added, at the end of each chapter, a section which discusses further results beyond the main focus of the text and open problems that we find challenging and important.

This book is addressed to both researchers and to graduate students with a good background in analysis and some previous exposure to partial differential equations. Both authors have used with success preliminary versions of the manuscript for second and third year graduate courses in pde.

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