Introduction

by B. Després

A common feature of all the works that have been done during the CEMRACS event is that they are concerned with modelization, discussion of the mathematical correctness of the models in relation with the physical foundations, and numerical methods for the approximate solution of the models. The difference is more in the balance of the three ingredients. However the success reached at the end is in all cases due to the fact that none of these three ingredients has been neglected, and that all three of them interact with the other two.

The GENJET project was proposed by G. Baudin and E. Lapébie (both from the DGA). The group was constituted with seven members, including C. Baranger, G. Baudin, L. Boudin, B. Despres, F. Lagoutière, E. Lapébie and T. Takahashi. The large number of participants was motivated by the importance of the task: essentially this project can be subdivided into a “hyperbolic” project and a “kinetic” project. The physical motivation was the modelization and numerical solution of the generation and break-up of droplets after the impact of a rigid body on a tank filled with a compressible fluid. As a consequence the physics runs from kinetic modelization of droplets to the coupling of a compressible fluid with a non deformable solid. In order to obtain substantial results for the duration of the CEMRACS, it has been decided to simplify the study. This is why both regimes are considered separately. In the kinetic part, a model of Reitz wave was preferred to the TAB (Taylor Analogy Break-up) model because the Weber number was relatively low. The conservativity of the model was checked: it gives a indication of its correctness. The numerical tests show qualitative agreement with the physics of droplets in this regime. The hyperbolic part of the study was about the coupling of a compressible fluid with a rigid body, but limited in dimension one. The method consists in the use of a mixed cell approach (i.e. an artificial multimaterial model) with an exact calculation of the balance of forces at the boundary between the fluid and the solid body. It gives quite satisfactory numerical results. As is usual for such problems, the conservation of total energy is not exact. More research has to be done, in particular for the multidimensional extension.

The MHYMOD project has concerned B. Despres, S. Jaouen, C. Mazeran and T. Takahashi. It was about numerical methods for multivelocity compressible modeling. The originality was the interpenetration of the fluid: at $t = 0$ an interface may exist that separates the different fluids; then due to some small scale phenomena, mixing and/or interpenetration occur. This study met many of the well-known difficulties inherent in this subject: possible loss of hyperbolicity, degeneracy of the model where one of the material/phase disappears, etc. One of the goals of the study was to propose new lagrangian numerical methods for this problem. Here lagrangian means that the scheme is two steps: in the first step the system is solved in the Lagrangian reference frame that moves with the average fluid; the second step is usually called remapping or projection. This study was also the occasion of a collaboration with S. Gavrilyuk (Université Marseille). It helped to understand the similarities between the multivelocity model that was at the base of the MHYMOD project and a model that appears in two fluids modelization in pipes. At the numerical level, the famous Ransom test case was solved with the new method with convincing results.

The DINMOD project has mobilized F. Caro, F. Coquel, D. Jamet and S. Kock around diffusive interface methods for two-phase flows modeling. Other projects of this CEMRACS event are about multiphase or multivelocity flows: this is still an active field of research, where modelization is perhaps half of the hard work. Let us recall that no all purpose multiphase or multivelocity partial differential equation model exists. On the contrary one has to adapt the modelization to the physical situation encountered. One difficulty is that many of the models, if not all, display strong deviations from the standard theory of hyperbolic systems of conservation laws. This motivates multidisciplinary research where modelization (with mechanical and thermodynamical aspects), the study of the well-posedness of the PDE model and the discretization issues interlace continuously. The study done in the DINMOD project is highly representative of this tendency. It begins with the modelization, making the parallel with grad-two fluids. The Hamilton least action principle is used to derive a model. The authors stress themselves that the use of the Hamilton principle has its own part of arbitrariness, even if most of the classical physics is based on it. Once the model has been established, the authors study the Mechanical Equilibrium together with the Thermodynamical Equilibrium of the model. It consists in studying various limits of the coefficients of relaxation. An important result is that these equilibriums are well posed. Here the well-posedness is understood in a very weak sense. Nevertheless it gives a sound basis that helps to design numerical methods based on a relaxation or projection method. The scheme is made of a classical hyperbolic stage, and of a less classical relaxation to equilibrium stage. Numerical results show the efficiency of the approach.

The work done by F. Coquel, D. Diehl, C. Merkle and C. Rhode is about the modelization and numerical capture of phase transitions solutions for liquid vapor flows, with an approach referred to as the sharp/diffusive interface method. Since the pioneering work of van der Walls, many researchers have realized that the solutions of these problems need an advanced modelization for which an appropriate mathematical theory is still to be completed. For instance most of the models display an elliptic instable region and the selection of shock solutions (by a selection principle) is needed to get uniqueness of the solution of the Riemann problem. Among all the contributions of this work, let us just mention that it was proposed to use the kinetic relation as the selection principle: all this is absolutely crucial at the numerical level. Examples are given to motivate the study: for instance spurious oscillations that one may encounter in bubble-in-a-liquid computations are related to the issue of the determination of the selection principle and of its application in numerical methods. A solution with the relaxation approach is proposed, with very nice numerical results. The method is based on an analysis that insures the decrease of the energy of the system. This study definitely collects many aspects of the current research about conservation laws for complex flows.

The work of J. Cartier and A. Munnier is motivated by the numerical solution of radiative transfer problems in view of Inertial Confinement Fusion (ICF) applications. The model problem is the radiative transfer equation for $I(x, t; \vec{\Omega})$ where $\vec{\Omega}$ is the direction of photons. The coupling with the matter leads to an integrodifferential equation. An important difficulty is to get a discretization compatible with the diffusion limit of the model. This diffusion limit is of parabolic type. This is also related to the well-known Eddington factor γ approximation. The flux limiter λ is another parameter that controls the physical quality of the diffusion approximation. This is based upon a work of Levermore that gives four conditions that γ and λ should respect to be compatible. A numerical comparison of some pairs (γ, λ) is made: the numerical results show a preference for the geometric model. However the authors stress in their conclusion that even with this approach the results are not completely satisfactory. This work is representative of the theory of diffusion approximation for some hyperbolic equations.

The work presented by M. Dumbser and C.-D. Munz is about arbitrary high order Discontinuous Galerkin schemes for linear and non linear hyperbolic systems of PDE's. Compressible gas dynamics is one example treated during the CEMRACS event. Recall that Discontinuous Galerkin Methods (DGM) is a very promising approach for the discretization of PDE's. The most important feature is probably that any order of accuracy is theoretically reachable on arbitrary grids even for complicated equations, provided that these equations can be written in divergence form. The particular DGM studied in this project is based on an idea of TORO: it consists in a Lax–Wendroff time integration, but at any order. Moreover the CPU time needed for the ADER scheme is lower with respect to standard Runge–Kutta time integration. This is important because it has been stressed by many researchers that reducing the cost of DGM is an issue. Of course it depends also on the implementation. The method proposed in this work gives a positive answer to the problem of reducing costs. Many numerical experiments show that the accuracy is also at the *rendez-vous*. Another class of applications that needs very high order schemes is long time simulation. This kind of problem appears in aeroacoustics. It requires to push standard methods to their limits in order to get satisfactory results. P^1 approximations are usually not enough. P^{10} is shown to be promising. This study is highly representative of the research done in the world to promote DGM for various hyperbolic problems. Some open points remain of course. For instance it seems difficult to merge the elegant mathematical presentation of the foundations of DGM with the limiter techniques that one needs to stabilize DG schemes. Those interested in all these approaches and issues will find a very complete presentation in this paper.

The work done by C.-D. Munz, M. Dumbser and M. Zucchini is about the modeling of Low Mach number flows and their numerical approximation. Apart from the report of the specific work that has been done at the CEMRACS, the paper gives a quite extended presentation of Low Mach number approximation of compressible gas dynamics. In the case $M = \frac{u_{\mathrm{ref}}}{c_{\mathrm{ref}}}$ is small, acoustic waves of long range and low energy may interact with small structures containing high energy. The idea of the method is to perform an asymptotic expansion in terms of the Mach parameter M. Different Ansatz leads to different models: many of them are very demanding at the numerical level. The method used in this work has recently been proposed by Klein and Munz: it is called the multiple pressure approximation. Numerical computations have been performed with this approach. For instance the acoustic pressure of a co-rotating vortex is computed using the ADER high order method described in another report of the CEMRACS 2003. An calculation is about the noise generated by a free jet. The method consists in feeding the ADER scheme for the computation of the acoustic waves with the data generated by a commercial software used to compute the large scale averaged flow. The results may be evaluated in view of the multiple pressure modelization. Quite impressive numerical results are provided. In summary this study is representative of new tendencies about the modelization and numerical approximation of the acoustic response of a compressible flow. We are convinced that this kind of work will inspire many others in the next years.