

Introduction

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Several projects were devoted to the numerical resolution of the Vlasov equation which is very important for the simulation of charged particle beams as well as globally neutral plasmas. The major difficulty involved is that the Vlasov equation is posed in phase space and thus the dimension is twice as high as for usual physical space simulations. For this reason Monte Carlo particle methods have had great success in this area. The coupling with the self-consistent electromagnetic field is done using the Particle-In-Cell (PIC) method. This technique is based on a Monte Carlo particle approximation of the Vlasov equation coupled with a grid based field solver in configuration space. Most simulation papers, in the last twenty years, involving charged particles have been performed using variants of this technique. Still, there are many problems, for which the method needs to be better understood and improved such as the coupling of the particle solution with a Maxwell solver, or the limitation of numerical noise in the simulations. The COCPIC project addresses the first of these issues and the PICONU project the second.

Other techniques for the numerical resolution of the Vlasov equation have been introduced for 1D problems in the seventies and have regained a new interest for multi-dimensional simulations recently with the increase of computer power. However, when going to 4D, 5D or 6D, phase space methods based on a uniform phase-space grid cannot compete with PIC methods except in some cases where very high precision is needed because of their computational complexity. In order to overcome this problem, adaptive phase-space grid methods have been introduced in the last years. Two such techniques were studied at CEMRACS 2003, one based on hierarchical finite elements and one based on interpolating wavelets. The first one is introduced in this book.

The work by Régine Barthelmé and Céline Parzani deals with the coupling of a Particle-In-Cell Vlasov solver with an FDTD Maxwell solver based on the Yee method. The underlying problem comes from the fact that the approximate charge and current densities calculated from the particles normally do not satisfy a discrete version of the continuity equation $\partial_t \rho + \nabla \cdot J = 0$. Because of this the computed electric field does not satisfy Gauss' law well enough on long time scales, which leads to very unphysical solutions. Several remedies have been proposed. They fall into two categories. The first consists in correcting the electric field from time to time in order that the error in Gauss' law does not become too large, and the second consists in computing the current density J from the particles such that the continuity equation

remains satisfied on the discrete level. This of course depends fundamentally on the field solver. Up to now two such methods had been introduced. The first one consists in computing the current using the actual fluxes of the numerical particles through the grid faces. This was introduced for the first order by Morse and Nielson and for the second order by Villasenor and Buneman. The other technique consists in computing J directly from the continuity equation by splitting between the different directions. This was introduced by Ezyrkepov at any order. The main contribution of the work in this book is first to give a better and clearer understanding of the Morse–Nielson and Villasenor–Buneman method and then to generalize the method to any order.

The paper by Chehab, Cohen, Jennequin, Nieto, Roland and Roche deals with the very important issue of noise reduction in Particle-In-Cell simulations. Many efforts have been made in this direction since this is the major problem of the method. These efforts include statistical techniques like variance reduction methods as in the variant which is called the δf method and which consists in applying the Particle-In-Cell method only to a perturbation of an equilibrium. This technique is very efficient but is only applicable in a limited set of problems where a known equilibrium, to which the solution stays close, exists. Apart from that, many filtering techniques applied generally in Fourier space have been introduced and are now available in most of the codes. In this work an original approach which consists in computing the charge density ρ in a well chosen wavelet basis and applying a thresholding procedure is introduced. This method is based on ideas introduced by Donoho in non parametric statistics and looks very promising for PIC simulations.

The work of Campos Pinto and Mehrenberger introduces adaptive grid based Vlasov solvers. The underlying numerical method is the semi-Lagrangian method which is based on the fact that the distribution function is conserved along characteristics. The method then consists in updating the grid values of the distribution function in two steps: 1) Find the origin of the characteristic curve ending at the different grid points. 2) Interpolate the distribution function at the origin of the characteristic curve from the known grid values. The adaptivity is introduced using non linear approximation. Indeed, using a hierarchical decomposition of the distribution function, it is possible to obtain an approximation, depending on the function being approximated, based on considerably fewer points. This technique allows to reduce the number of needed interpolation points compared to a uniform fine grid while increasing the error only slightly and in a controlled manner.

The paper by Campos Pinto and Mehrenberger investigates non linear approximation based on hierarchical bi-quadratic finite elements. This approach has the advantage of being cell based and completely local, which makes it far easier to parallelize.

The contribution of Crouseilles and Filbet is devoted to the derivation of a high order scheme for the simulation of a collisional plasma. The model they consider is the Vlasov equation with an additional Fokker–Planck–Landau collision operator. The numerical scheme decouples the transport part from the collision part. A new second order method for the Vlasov equation is derived which conserves mass and

energy. Slope limiters are introduced to ensure positivity of the distribution function. However, when those are needed, energy conservation is no more exactly verified. On the other hand the collision operator is discretized using a spectral method. The numerical scheme is validated in particular on the Landau damping problem which is thoroughly discussed and which should be of great importance to people interested in code validation examples.

The project by C. Besse, N. Mauser and H. P. Stimming is concerned with numerical experiments of a time splitting strategy to solve two non linear models arising in different physical situations. The first is the so-called Schrödinger–Poisson- X_α (SPX_α) model. This model arises in the modeling of quantum particle dynamics and is intended to be an approximation of the time dependent Hartree–Fock equation. The second is the Davey–Stewartson (DS) model, which is motivated by the description of water waves. On the one hand, SPX_α involves two nonlinearities: one is purely local and is nothing but the usual focusing cubic nonlinearity in the Schrödinger equation, while the other is non local, since it is related to self-consistent interactions. The behavior of the solutions, in the semi classical limit $\varepsilon \rightarrow 0$, ε being defined from the Planck constant, highly depends on the relative strength of both nonlinearities, scaled in an ε -dependent way. Addressing the problem from a numerical viewpoint is a really challenging issue: using a classical finite difference approach would require unrealistic restrictions on the mesh size. On the other hand, DS involves a set of parameters, and depending on the range of the values of these parameters, the equation changes type. In turn, the solutions of DS can exhibit a very rich behavior, like formation of soliton structures, or blowup phenomena. Therefore, the numerical challenge is the precise evaluation of time and space localization of the blowup and the description of the profiles with sharp accuracy. Here the authors adapt a time splitting spectral method introduced by Bao, Jin and Markowich. As usual, the idea is to separate the problem by first solving a linear PDE, and then dealing with a nonlinear ODE, possibly stiff, which can be solved exactly. A proof of convergence of the semi-discrete scheme is proposed, and simulations illustrate the behavior of the solutions and the accuracy of the method by various multidimensional examples.

The contribution by C. Besse, P. Degond, F. Deluzet, J. Claudel, G. Gallice, and T. Tessieras is devoted to modeling issues for ionospheric plasmas, a subject of crucial importance for communication satellites. The models in this field can exhibit instability phenomena and strong heterogeneities can occur, a situation comparable to Rayleigh–Taylor instabilities in fluid mechanics. The basic model which describes ionospheric plasmas couples Euler equations for ions and electrons, and Maxwell equations for the evolution of electric and magnetic fields. The computational cost to solve numerically such a complicated system is prohibitive when dealing with physical values of the parameters, which motivates that one seeks for simplified models. Hence, having identified a set of dimensionless relevant parameters, a hierarchy of models is presented, which ends with the so-called striation model. This model involves a 3D convection diffusion equation coupled with a 2D elliptic equation. However, the stability analysis reveals that the striation does not reproduce the main features of the

physics: dissipation phenomena have been destroyed in the asymptotics. Then, the idea consists in taking into account turbulence effects in the fluid evolution, which introduces an additional eddy viscosity and restores nice dissipation properties. In turn, the new “turbulent striation model” presents better stability properties. Numerical experiments validate the modeling discussion.

The paper by L. Gosse is a review of different numerical and asymptotic methods for high frequency asymptotics of the 1D Schrödinger equation.

Four techniques for dealing with such asymptotics are briefly presented: stationary phase, Wigner measure, WKB, ray tracing. Two numerical methods based on moment closure are described (the classical moment system and the K branch entropy solution) and are illustrated by a classical example.