

Preface

The aim of this book is to present Kervaire's work on differentiable knots in higher dimensions in codimension $q = 2$. As explained in our introduction (see Section 1.7), this book is written to make the reading of papers by Michel Kervaire and Jerome Levine easier. Thus we hope to communicate our enthusiasm for higher-dimensional knot theory. In order to appreciate the importance of Kervaire's contribution, we describe in Chapters 2 to 4 what was, at the time, the situation in differential topology and in knot theory in codimension $q \geq 3$. In Chapter 5, we expose Kervaire's characterization of the fundamental group of a knot complement. In Chapter 6, we explain Kervaire and Levine's work on knot modules. In Chapter 7, we detail Kervaire's construction of the "simple knots" classified by Jerome Levine. Chapter 8 summarizes Kervaire and Levine's results on knot cobordism. In Chapter 9, we apply higher-dimensional knot theory to singularities of complex hypersurfaces. Appendixes A to D are devoted to a discussion of some basic concepts, known to the experts: signs, Seifert hypersurfaces, open book decompositions and handlebodies. In Appendix E, we conclude with an exposition of the results of Hill–Hopkins–Ravenel on the Kervaire invariant problem and its consequences for knot theory in codimension 2.

The point of view adopted here is somewhat pseudo-historical. When we make explicit Kervaire's work we try to follow him closely, in order to retain some of the flavor of the original texts. When necessary we add further contributions, often due to Levine. We also propose developments that occurred later.

We thank Peter Landweber for patiently correcting our English. The figures in Appendix F are due to Cam Van Quach. We thank her for her friendly collaboration.

Françoise Michel
Claude Weber
Geneva, September 2016