Cavicchioli, Alberto; Hegenbarth, Friedrich; Repovš, Dušan

Higher-dimensional generalized manifolds: surgery and constructions. (English)

This book grew out of the authors’ studies of the work of Bryant, Ferry, Mio and Weinberger on generalized manifolds. A space \( X \) is said to be a geometric generalized \( n \)-manifold if it satisfies the following two properties (i) \( X \) is a Euclidean neighborhood retract and (ii) \( X \) is a homology \( n \)-manifold, that is for every \( x \in X \), \( H_*(X, X\setminus\{x\}; \mathbb{Z}) \cong H_*(\mathbb{R}^n, \mathbb{R}^n\setminus\{0\}; \mathbb{Z}) \). Homology manifolds are thus somewhere between manifolds and Poincaré complexes. An important question that one can ask about a generalized \( n \)-manifold is whether it has a resolution, that is to say does there exist a proper cell-like map \( f : M \to X \) from a topological \( n \)-manifold \( M \) onto \( X \). One of the main results discussed in the book due to F. Quinn [Mich. Math. J. 34, 285–291 (1987; Zbl 0652.57011)] is a resolution theorem for dimension \( n \geq 5 \) that defines an integer valued invariant \( I(X) \) in \( H_0(X; \mathbb{Z}) \) – the Quinn Index – such that \( I(X) \equiv 1 \) (mod 8). A resolution exists if \( I(X) = 1 \). After a chapter that introduces generalized manifolds and resolutions the authors turn to a review of surgery theory. Motivated by the aim of defining the surgery exact sequence for homology manifolds, which lack nice local geometric properties, they are particularly concerned to define the surgery spectra and derive the surgery exact sequence before moving on to considering bounded surgery as an instance of controlled surgery. A basic idea in performing controlled surgery is to work on the level of chain complexes because this allows one to keep the surgeries small. Chapter 3 of the book applies controlled surgery methods to construct generalized manifolds. The authors review Quinn’s work and the theorems of J. Bryant et al. [Bull. Am. Math. Soc., New Ser. 28, No. 2, 324–328 (1993; Zbl 0799.57014); Geom. Topol. 11, 1289–1314 (2007; Zbl 1144.57019)]. The final chapter addresses the question of generalized manifolds and surgery theory. The goal is to obtain a version of the surgery exact sequence. The obstacle here is that homology manifolds unlike manifolds do not have nice local geometrical properties, therefore a geometric approach to a surgery exact sequence in not available. The methods for its construction must rely on algebraic arguments using the Quinn invariant.

The book covers a large amount of material in detail while also referencing other sources for some details. The result is a somewhat challenging book that nonetheless provides an excellent introduction to difficult material.

Reviewer: Jonathan Hodgson (Swarthmore)

MSC:

- 57-02 Research monographs (manifolds)
- 57P05 Local properties of generalized manifolds
- 57P10 Poincaré duality spaces
- 57R65 Surgery and handlebodies
- 57R67 Surgery obstructions, Wall groups
- 57N15 Topology of the Euclidean \( n \)-space, \( n \)-manifolds (\( 4 \leq n \leq \infty \))
- 57N65 Algebraic topology of manifolds

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homology manifold; Poincaré duality; degree 1 normal map; boundedly controlled surgery; surgery spectrum; assembly map; Quinn index; Euclidean neighborhood retract; cell-like resolution; disjoint disks property; manifold recognition problem

Full Text: DOI