Preface

This book deals with the local refinement or Morreyfication $\mathcal{L}^r A^s_{p,q}(\mathbb{R}^n)$ of $A^s_{p,q}(\mathbb{R}^n)$. Here $A^s_{p,q}(\mathbb{R}^n)$ stands for the nowadays well-known scales of spaces $B^s_{p,q}(\mathbb{R}^n)$ and $F^s_{p,q}(\mathbb{R}^n)$ covering (fractional) Sobolev spaces, (classical) Besov spaces and Hölder–Zygmund spaces, whereas $\mathcal{L}^r$ is intended to remind the reader of the Morrey–Campanato spaces $\mathcal{L}^r_p(\mathbb{R}^n)$. The history of these spaces, $A^s_{p,q}$ on the one hand, and $\mathcal{L}^r_p$ on the other, began in the mid 1930s, both being closely connected with linear and nonlinear PDEs. But these rather different types of spaces merged only in the last 10–15 years. We present a new approach based on the elaborated theory of the spaces $A^s_{p,q}(\mathbb{R}^n)$ as it stands now. We assume that the reader has a working knowledge about basic assertions for the spaces $A^s_{p,q}(\mathbb{R}^n)$. But to make this book independently readable we provide related notation, facts, and detailed references. We prove some new more specific assertions for $A^s_{p,q}(\mathbb{R}^n)$. Otherwise we develop in Chapters 1 and 2 the theory of the spaces $\mathcal{L}^r A^s_{p,q}(\mathbb{R}^n)$ in detail. In Chapter 3 we introduce the Morrey–Campanato spaces $\mathcal{L}^r_p(\mathbb{R}^n)$ and compare them with $A^s_{p,q}(\mathbb{R}^n)$ and $\mathcal{L}^r A^s_{p,q}(\mathbb{R}^n)$. Chapter 4 is a self-contained introduction to Gagliardo–Nirenberg inequalities both in the global spaces $A^s_{p,q}(\mathbb{R}^n)$ and in the local spaces $\mathcal{L}^r A^s_{p,q}(\mathbb{R}^n)$. We apply the theory of the local spaces $\mathcal{L}^r A^s_{p,q}(\mathbb{R}^n)$ and (to a minor extent) also of the global spaces $A^s_{p,q}(\mathbb{R}^n)$ in Chapter 5 to (linear and nonlinear) heat equations and in Chapter 6 to Navier–Stokes equations.

Formulae are numbered within chapters. Furthermore in each chapter all definitions, theorems, propositions, corollaries and remarks are jointly and consecutively numbered. Chapter n is divided in sections $n.k$ and subsections $n.k.l$. But when quoted we refer simply to Section $n.k$ or Section $n.k.l$ instead of Section $n.k$ or Subsection $n.k.l$. References are ordered by names, not by labels, which roughly coincide, but may occasionally cause minor deviations. The numbers behind the items in the Bibliography mark the page(s) where the corresponding entry is quoted. All unimportant positive constants will be denoted by c (with additional marks if there are several c's in the same formula). To avoid any misunderstanding we fix our use of $\sim$ (equivalence) as follows. Let I be an arbitrary index set. Then

$$a_i \sim b_i \quad \text{for } i \in I \quad \text{(equivalence)}$$

for two sets of positive numbers $\{a_i : i \in I\}$ and $\{b_i : i \in I\}$ means that there are two positive numbers c_1 and c_2 such that

$$c_1 \, a_i \le b_i \le c_2 \, a_i \quad \text{for all } i \in I.$$

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Jena, Spring 2013 Hans Triebel