Introduction

The quest for understanding the structure of algebraic varieties is a very natural one. In the 19th century it was realized that the birational classification of algebraic curves consists in two steps: an irreducible curve has a unique smooth model and its genus g is a discrete invariant, and for fixed genus g the isomorphism classes of curves depend (for $g > 1$) on $3g - 3$ parameters, the moduli. Classification of varieties goes back to the attempt of the Italian school around the turn of the 20th century to extend this to dimension 2 and to understand algebraic surfaces. Castelnuovo indicated how one could obtain a minimal model of a surface and showed that the model is unique if the surface is not ruled. Armed with this, Enriques set up a rough classification of surfaces that was based on the behaviour of the pluri-canonical systems. Deservedly it is viewed as the main achievement of the Italian School of algebraic geometry around the beginning of the 20th century.

In the 1960s Kodaira re-examined this classification and put it in a modern framework using the newly obtained techniques of algebraic and complex analytic geometry. He extended the classification to compact complex analytic surfaces. The work of Kodaira was continued by Kawai, Iitaka, Ueno and others. A landmark from that time was Hironaka's 1964 theorem on the existence of a smooth model for any complex algebraic variety.

The case of surfaces suggested to look for nefness of the canonical class as a guiding principle. But it was soon realized that the theory in higher dimensions is much more intricate than in dimension 2. The reason was the existence of smooth varieties not possessing a smooth model with a nef canonical class. Thus one had to deal with singularities. It slowly emerged in the 1970s and the early 1980s in work of Reid, Kawamata, Mori and others that by restricting the type of singularities that minimal models were allowed to have one could still build a useful theory of such models. The Minimal Model Program is the quest for a minimal or simplest projective variety that is birational to a given variety and is based on the expectation that there exists a model with limited singularities and nef canonical class or a morphism to a lower-dimensional variety with ample relative anti-canonical class. Contrary to dimension 2, contractions alone should not suffice to reach a minimal model, but other birational transformations, the flips, are required.

The work of Mori in the 1980s analyzed threefolds for which the canonical bundle is not nef. An important role was played by Mori's cone of curves, based on his famous Bend and Break theorem that he had developed in 1979. Shokurov showed the termination of flips in the three-dimensional case and in 1988 Mori completed this by his theorem on the existence of flips for three-dimensional varieties, using earlier work of Kawamata and Tsunoda. In this way the combined efforts of Reid, Mori, Kawamata, Kollár, Shokurov and others led to a generalization of the work of the Italian school to dimension 3.

The existence of flips and the termination of the process formed the main obstacles for progress in arbitrary dimension. The existence of flips is closely linked to the finite generation of the canonical ring.

From 2005 onwards, great progress has been achieved, first by Hacon and McKernan, whose work was then complemented by Birkar and Cascini. Besides the existence of flips in arbitrary dimension using techniques of Shokurov and Siu, this has led to the cornerstone result that for any smooth projective variety the canonical ring is finitely generated.

It was this wave of breakthroughs and surprising new developments that motivated us to devote a conference to the theme "Classification of Varieties." The conference can be seen as a successor of the Texel conferences, though it did not take place on Texel Island, but on another island, Schiermonnikoog. It was held in the second week of May 2009. The present volume is published on the occasion of the conference, although most contributions are not related to lectures given at the conference. It gives ample evidence of the progress that is now being made.

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