

Contents

Preface	vii
1 Prerequisites	1
2 Algebraic Sets, Morphisms, and Rational Maps	12
2.1 Review of topology	12
2.2 The correspondences V and I	15
2.3 Morphisms	21
2.4 Rational maps	24
2.5 Projective algebraic sets	30
2.6 Rational maps and birational equivalence	35
2.7 Complements and exercises	43
3 Geometric Properties of Algebraic Varieties	51
3.1 Tangent space, singularities and dimension	51
3.2 Independence of polynomials. Essential parameters	62
3.3 Dimension of a projective variety	68
3.4 Order of a projective variety, tangent cone and multiplicity	72
4 Rudiments of Elimination Theory	83
4.1 Resultant of two polynomials	83
4.2 Bézout's theorem for plane curves	89
4.3 More on intersection multiplicity	90
4.4 Elimination of several variables	99
4.5 Bézout's theorem	102
5 Hypersurfaces in Projective Space	106
5.1 Generalities on hypersurfaces	106
5.2 Multiple points of a hypersurface	108
5.3 Algebraic envelopes	115
5.4 Polarity with respect to a hypersurface	119
5.5 Quadrics in projective space	126
5.6 Complements on polars	134
5.7 Plane curves	140
5.8 Surfaces in \mathbb{P}^3	150

6	Linear Systems	166
6.1	Linear systems of hypersurfaces	167
6.2	Hypersurfaces of a linear system that satisfy given conditions . . .	168
6.3	Base points of a linear system	170
6.4	Jacobian loci	177
6.5	Simple, composite, and reducible linear systems	182
6.6	Rational mappings	185
6.7	Projections and Veronese varieties	189
6.8	Blow-ups	192
7	Algebraic Curves	197
7.1	Generalities	197
7.2	The genus of an algebraic curve	201
7.3	Curves on a quadric	212
7.4	Rational curves	218
7.5	Exercises on rational curves	225
8	Linear Series on Algebraic Curves	238
8.1	Divisors on an algebraic curve with ordinary singularities	239
8.2	Linear series	246
8.3	Linear equivalence	248
8.4	Projective image of linear series	251
8.5	Special linear series	256
8.6	Adjoints and the Riemann–Roch theorem	261
8.7	Properties of the canonical series and canonical curves	270
8.8	Some results on algebraic correspondences between two curves . .	274
8.9	Some remarks regarding moduli	277
8.10	Complements and exercises	284
9	Cremona Transformations	292
9.1	Quadratic transformations between planes	292
9.2	Resolution of the singularities of a plane algebraic curve	297
9.3	Cremona transformations between planes	307
9.4	Cremona transformations between projective spaces of dimension 3	317
9.5	Exercises	322
10	Rational Surfaces	340
10.1	Planar representation of rational surfaces	340
10.2	Linearly normal surfaces and their projections	349
10.3	Surfaces of minimal order	354
10.4	The conics of a plane as points of \mathbb{P}^5 and the Veronese surface . .	360
10.5	Complements and exercises	364

11 Segre Varieties	386
11.1 The product of two projective lines	386
11.2 Segre morphism and Segre varieties	389
11.3 Segre product of varieties	392
11.4 Examples and exercises	395
12 Grassmann Varieties	399
12.1 The lines of \mathbb{P}^3 as points of a quadric in \mathbb{P}^5	399
12.2 Complexes of lines in \mathbb{P}^3	403
12.3 Congruences of lines in \mathbb{P}^3	407
12.4 Ruled surfaces in \mathbb{P}^3	408
12.5 Grassmann coordinates and Grassmann varieties	414
12.6 Further properties of $\mathbb{G}(1, n)$ and applications	422
13 Supplementary Exercises	433
13.1 Miscellaneous exercises	433
13.2 Further problems	454
13.3 Exercises on linear series on curves	457
Bibliography	467
Index	475