Addenda: Complete Boolean algebras of type I factors

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Lemmas 4. 4, 4. 6 and a slightly strengthened version of 4. 7 can be proved in one step as follows.

**Lemma.** Let \{R_a\}_a \in A be a type I factorization. If there exists a vector \(\psi\) such that

\[
\inf \text{d}(\psi; R_{a_1}, \ldots R_{a_n}, R(\{\alpha_1, \ldots \alpha_n\})) = \varepsilon > 0
\]

where \(\inf\) is taken over all \(n, \alpha_1, \ldots \alpha_n\), then \(R_a\) is a TPF.

**Proof.** For each finite \(J \subset A\) choose a minimal projection \(P_j^{(\psi)} \in R_j\) for all \(j \in J\) such that

\[
(\psi, \Pi_{j \in J} P_j^{(\psi)} \psi) \geq \varepsilon.
\]

Let

\[
\psi(J) = \Pi_{j \in J} P_j^{(\psi)} \psi,
\]

\[
S(J) = \bigcup_{K \subset J} \psi(K),
\]

\[
S = \bigcap_j S(J)^{\omega y}.
\]

Let \(\Phi \in S\). By lemma 4.1 we have \((\psi, \Phi) \geq \varepsilon\). For any \(j \in J\), all vectors in \(S(J)\) are product vectors in

\[
H = H_j \otimes H_j',
\]

where

\[
R_j = B(H_j) \otimes 1.
\]

By lemma 6.2 any vector in \(S(J)^{\omega y}\) is a limit of a sequence of ele-
ments in \( S(J) \). By lemma 3.3 \( \emptyset \in \mathcal{S} \) is thus a product vector. Hence there exists a minimal projection \( P_f \in R_f \) such that \( P_f \emptyset = \emptyset \). Hence \( \emptyset \) is factorizable and \( R_a \) is a TPF by lemma 4.3. Q. E. D.