An endpoint estimate for some maximal operators

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Suppose $\mu$ is a finite positive Borel measure on $\mathbb{R}^n$. It is proved in [DR] that if the Fourier transform of $\mu$ satisfies a decay estimate

\begin{equation}
|\hat{\mu}(\xi)| \leq C|\xi|^{-\alpha}
\end{equation}

for some $\alpha > 0$, then the maximal operator

\begin{equation}
Mf(x) = \sup_{k \in \mathbb{Z}} \int |f(x - 2^k y)| \, d\mu(y)
\end{equation}

is bounded on $L^p(\mathbb{R}^n)$ for $1 < p < \infty$. On the other hand, Theorem 4 in [C2] states that if $\mu$ is the Lebesgue measure $\sigma_{n-1}$ on the unit sphere $\Sigma_{n-1}$ in $\mathbb{R}^n$, then (2) maps $H^1(\mathbb{R}^n)$ into $L^{1,\infty}(\mathbb{R}^n)$. The purpose of this paper is to adapt the method of [C2] to prove an $H^1-L^{1,\infty}$ result for (2) requiring, in the spirit of [DR], only a certain decay of $\hat{\mu}$.

**Theorem.** Suppose $\mu$ is a finite positive Borel measure on $\mathbb{R}^n$ with support in $[-1,1]^n$. If

\begin{equation}
|\hat{\mu}(\xi)| \leq C|\xi|^{-n/2},
\end{equation}

then (2) maps $H^1(\mathbb{R}^n)$ into $L^{1,\infty}(\mathbb{R}^n)$.

As indicated, our proof follows the method of proof of Theorem 4 of [C2]. Our view is that the interest of this paper lies as much in a
demonstration of the flexibility of that method (see [C2, Remark 7.2]) as in our result. Although many of the details differ, the main novelty here lies in the use of the auxiliary functions $\varphi_N$ to handle the control (see (7)) of

$$\left\| \left( \sum_{\kappa(Q) = j-s} \lambda_Q a_Q \right) * \mu_j \right\|_2.$$

The proof in [C2] used the curvature of the support of $\sigma_{n-1}$ in the analogous estimate. Our argument proceeds, albeit in the same spirit, with no knowledge of $\mu$ aside from the decay of $\hat{\mu}$. But we pay by requiring a higher rate of decay $-\sigma_{n-1}(\xi)$ decays, as is well-known, like $|\xi|^{(1-n)/2}$. Still, there exist singular measures on $\mathbb{R}^n$ satisfying our hypothesis. (This was proved in [I-M] for $n = 1$ - see Lemma 1 [K, p. 165] for the extension from Fourier coefficients to Fourier transform. To get a singular measure $\mu$ on $\mathbb{R}^n$ with $|\hat{\mu}(\xi)| = O(|\xi|^{-n/2})$, let $\nu$ be the measure from [I-M] translated to have support in $[1,2]$ and define the measure $\mu$ on $\mathbb{R}^n$ by

$$\int_{\mathbb{R}^n} f \, d\mu = \int_1^2 \int_{\Sigma_{n-1}} f(ry) d\sigma_{n-1}(y) r^{(n-1)/2} d\nu(r).$$

Then asymptotic estimates for Bessel functions such as those in [SW, Lemma 3.11] combine with the decay of $\hat{\nu}$ to give $|\hat{\mu}(\xi)| = O(|\xi|^{-n/2})$.)

It may be that our $n/2$ can be replaced by smaller $\alpha > 0$, thus yielding a more satisfying endpoint analog of the result of [DR]. The referee has pointed out that the paper [S] contains a point of similarity to the proof of our theorem (in its use of the Fourier transform for the $L^2$ estimate) and that ideas equivalent to some of those in [DR] are present in [C1]. We begin with two lemmas.

**Lemma 1.** For any $\alpha > 0$ and any finite collection of dyadic cubes $Q \subseteq \mathbb{R}^n$ and associated positive scalars $\lambda_Q$, there exists a collection $\mathcal{S}$ of pairwise disjoint dyadic cubes $S$ such that

a) $\sum_{Q \subseteq S} \lambda_Q \leq 2^n \alpha |S|$, if $S \in \mathcal{S}$,

b) $\sum |S| \leq \alpha^{-1} \sum \lambda_Q$,

c) $\left\| \sum_{Q \text{ not contained in any } \mathcal{S}} \lambda_Q |Q|^{-1} \chi_Q \right\|_{\infty} \leq \alpha$. 


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Proof. In the proof of Lemma 4.1 of [C2], simply replace 8 by $2^n$ and interpret dyadic in the $n$-dimensional Euclidean sense (instead of the parabolic sense in $\mathbb{R}^2$).

Notation. If $Q$ is a dyadic cube in $\mathbb{R}^n$ with side-length $2^j$, write $\sigma(Q)$ to stand for $j$. If $\sigma \in \mathbb{Z}$, let $\mathcal{R}_\sigma$ be the collection of dyadic cubes $Q \subseteq \mathbb{R}^n$ with $\sigma(Q) = \sigma$. Finally, if $Q \in \mathcal{R}_\sigma$, define $Q^* = Q + [-2^\sigma, 2^\sigma]^n$. Thus $Q^*$ is the union of $3^n$ cubes in $\mathcal{R}_\sigma$.

Lemma 2. (cf. [C2, Lemma 5.1]) Suppose given the following: some $\alpha > 0$, a collection $\mathcal{S}$ of pairwise disjoint dyadic cubes $S \subseteq \mathbb{R}^n$, a finite collection $\mathcal{C}$ of dyadic cubes $Q \subseteq \mathbb{R}^n$ such that each $Q \in \mathcal{C}$ is contained in some $S = S(Q) \in \mathcal{S}$, and for each $Q \in \mathcal{C}$ a positive number $\lambda_Q$. Then there exist a measurable $E \subseteq \mathbb{R}^n$ and a function $\kappa : \mathcal{C} \to \mathbb{Z}$ such that

a) $|E| \leq 3^n(\alpha^{-1} \sum \lambda_Q + \sum |S|)$,

b) $Q + [-2^j, 2^j]^n \subseteq E$, if $j < \kappa(Q)$ and $Q \in \mathcal{C}$,

c) $\sigma(S(Q)) < \kappa(Q)$ (for any $Q \in \mathcal{C}$),

d) for $\sigma \in \mathbb{Z}$ any $q \in \mathcal{R}_\sigma$, $\sum_{Q \subseteq q, \kappa(Q) \leq \sigma} \lambda_Q \leq \alpha 2^{n(\sigma+1)}$.

Proof. The proof is an adaptation of (and simpler than) that of Lemma 5.1 in [C2]. But we give the details for completeness and for the convenience of the reader.

Let $m = \min\{\sigma(Q)\}$. Find $\sigma_0 \in \mathbb{Z}$ such that
\[ \sum \lambda_Q < \alpha 2^{m\sigma_0}, \quad \sigma_0 > \max\{\sigma(Q)\}. \]

The proof is a stopping time argument on the descending parameter $\sigma$ and proceeds by dividing $\mathcal{C}$ into disjoint subcollections $\mathcal{C}_1$ and $\mathcal{C}_2$. We begin with $\sigma = \sigma_0 - 1$ and define, for $q \in \mathcal{R}_\sigma$,
\[ \Lambda_\sigma(q) = \sum_{Q \subseteq q} \lambda_Q. \]

Say that $q \in \mathcal{R}_\sigma$ is “selected at step $\sigma$” if
\[ \Lambda_\sigma(q) > \alpha 2^{n\sigma}. \]

Put into $\mathcal{C}_1$ every $Q$ such that $Q \subseteq q$ for some $q$ selected at step $\sigma$, and for such $Q$ define
\[ \kappa(Q) = \max\{1 + \sigma, 1 + \sigma(S(Q))\}. \]
Next, put into \( C_2 \) every \( Q \in \mathcal{C} \sim C_1 \) such that \( \sigma(Q) > \sigma \) - such a \( Q \) will actually satisfy \( \sigma(Q) = \sigma + 1 \) - and for such \( Q \) define

\[
\kappa(Q) = 1 + \sigma(S(Q)) .
\]

Note that (3) and (4) guarantee that (c) holds. Now replace \( \sigma \) by \( \sigma - 1 \) and repeat the process with

\[
\Lambda_\sigma(q) = \sum_{Q \subseteq q} \lambda_Q = \sum_{\substack{Q \subseteq q \\subseteq C_1 \\cup C_2}} \lambda_Q , \quad q \in \mathcal{R}_\sigma .
\]

(The last equality holds because \( Q \in C_2 \) at the beginning of step \( \sigma \) implies \( \sigma(Q) \geq \sigma + 2 \).) After the step \( \sigma = m \) we will have \( \mathcal{C} = C_1 \cup C_2 \) and \( \kappa \) defined on all of \( \mathcal{C} \). Next define

\[
E_1 = \bigcup_{q \text{ selected}} q^* , \quad E_2 = \bigcup S^* , \quad E = E_1 \cup E_2 .
\]

Then, since distinct selected \( q \) are disjoint,

\[
|E_1| \leq 3^n \sum_{q \text{ selected}} 2^{n\sigma(q)} < \frac{3^n}{\alpha} \sum_{q \text{ selected}} \Lambda_\sigma(q) \leq \frac{3^n}{\alpha} \sum \lambda_Q .
\]

Now a) follows since \( |S^*| = 3^n |S| \).

If \( \kappa(Q) = 1 + \sigma(S(Q)) \) and if \( j < \kappa(Q) \), then

\[
Q + [-2^j, 2^j]^n \subseteq S^* \subseteq E_2 .
\]

If \( \kappa(Q) \neq 1 + \sigma(S(Q)) \), then \( Q \subseteq q \) for some \( q \) selected at some step \( \sigma \) and \( \kappa(Q) = 1 + \sigma(q) \). Thus if \( j < \kappa(Q) \),

\[
Q + [-2^j, 2^j]^n \subseteq E_1 .
\]

So b) is verified.

Finally, if \( q \in \mathcal{R}_\sigma \) for \( \sigma \geq \sigma_0 - 1 \), then d) is clear from the choice of \( \sigma_0 \). So suppose \( \sigma < \sigma_0 - 1 \) and \( q \in \mathcal{R}_\sigma \). Now

\[
\Lambda_\sigma(q) \leq \alpha 2^{n(\sigma + 1)}
\]

or else the \( q_1 \in \mathcal{R}_{\sigma + 1} \) that contains \( q \) would have been selected at stage \( \sigma + 1 \). Since \( \kappa(Q) \leq \sigma \) implies that \( Q \not\in C_1 \) at the beginning of step \( \sigma \),

\[
\sum_{Q \subseteq q} \lambda_Q \leq \Lambda_\sigma(q) ,
\]
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Now suppose $\mu$ is a positive Borel probability measure supported on $[-1,1]^n$ and satisfying $|\hat{\mu}(\xi)| \leq C|\xi|^{-n/2}$. Let $f \in H^1(\mathbb{R}^n)$ have the form of a finite sum

$$f = \sum \lambda_Q a_Q,$$

where $\lambda_Q > 0$ and $a_Q$, supported in a cube $Q$, satisfies

$$\|a_Q\|_\infty \leq |Q|^{-1}, \quad \int_Q a_Q = 0.$$

As in [C2], a device of Garnett and Jones involving auxiliary dyadic grids allows us to assume that each $Q$ is dyadic. Fix $\alpha > 0$. It is enough to show that

$$\|Mf > 2\alpha\| \leq \frac{C}{\alpha} \sum \lambda_Q,$$

where $C$ depends only on $\mu$ and $n$.

Following [C2], let $\mathcal{S}$ be as in Lemma 1 and define

$$b = \sum_{s \in \mathcal{S}} \sum_{Q \subseteq s} \lambda_Q a_Q, \quad g = f - b.$$

Then $\|g\|_\infty \leq \alpha$ from Lemma 1.c) and so $|Mg| \leq \alpha$ (because $\mu$ has mass 1). Thus (5) will follow from

$$\|Mb > \alpha\| \leq \frac{C}{\alpha} \sum \lambda_Q.$$

Now, with $\mathcal{S}$ as above and with $\mathcal{C}$ the collection of $Q$'s appearing in the definition of $b$, let $\kappa$ and $E$ be as in Lemma 2. Since $|E| \leq C\alpha^{-1} \sum \lambda_Q$, it is enough to prove

$$\|Mb\|_{L^2(\mathbb{R}^n \sim E)}^2 \leq C\alpha \sum \lambda_Q.$$

Let $\mu_j$ be the dilate of $\mu$ defined by

$$\langle \varphi, \mu_j \rangle = \int_{\mathbb{R}^n} \varphi(2^j x) \, d\mu(x)$$
so that $\mu_j$ is supported in $[-2^j, 2^j]^n$ and
\[
Mb(x) = \sup_{j \in \mathbb{Z}} |b * \mu_j(x)|.
\]

If $Q \in C$, then, by Lemma 2.b), $a_Q * \mu_j$ is supported in $E$ unless $j \geq \kappa(Q)$. Thus if $x \notin E$,
\[
|M b(x)|^2 \leq \sum_j |b * \mu_j(x)|^2
\]
\[
= \sum_j \left( \sum_{\kappa(Q) \leq j} |\lambda_Q a_Q| \right) \mu_j(x)^2
\]
\[
= \sum_j \sum_{s=0}^{\infty} \left( \sum_{\kappa(Q) = j-s} \lambda_Q a_Q \right) \mu_j(x)^2.
\]
So, for $x \notin E$
\[
|M b(x)| \leq \sum_{s=0}^{\infty} \left( \sum_j \left( \sum_{\kappa(Q) = j-s} \lambda_Q a_Q \right) \mu_j(x)^2 \right)^{1/2}.
\]

Now (6) will follow from
\[
\left\| \left( \sum_j \left( \sum_{\kappa(Q) = j-s} \lambda_Q a_Q \right) \mu_j \right)^2 \right\|_2 \leq C \alpha(s + 1) 2^{-s} \sum \lambda_Q
\]
and so from
\[
(7) \quad \left\| \left( \sum_{\kappa(Q) = j-s} \lambda_Q a_Q \right) \mu_j \right\|_2^2 \leq C \alpha(s + 1) 2^{-s} \sum_{\kappa(Q) = j-s} \lambda_Q.
\]
The proof of (7) requires another lemma.

**Lemma 3.** For $N = 1, 2, \ldots$, there exist functions $\varphi_N \in L^1(\mathbb{R}^n)$ such that

a) $|\hat{\varphi}_N(\xi)| \geq (1 + |\xi|)^{-n/2}/C$, if $|\xi| \leq N - 1$. 
b) \(|\hat{\varphi}_N(\xi)| \leq C|\xi|^{-n/2}\),
and if \(L_N = \varphi_N * \hat{\varphi}_N \ (\hat{\varphi}_N(x) = \varphi_N(-x))\), then
c) \(\text{supp}(L_N) \subseteq [-1, 1]^n\),
d) \(|L_N(x) - L_N(y)| \leq C|x - y|/\min\{|x|, |y|\}.

**Proof.** We will construct \(L_N\) first and then \(\varphi_N\). Define \(h_N \in C(\mathbb{R}^n)\) by
\[
\hat{h}_N(\xi) = \begin{cases} 
1, & \text{if } |\xi| \leq 1, \\
|\xi|^{-n}, & \text{if } 1 < |\xi| \leq N, \\
0, & \text{if } |\xi| > N.
\end{cases}
\]
Choose a radial function \(\rho \in C_c^\infty(\mathbb{R}^n)\) such that
\[
\int \rho = 1, \quad \text{supp}(\rho) \subseteq [-1, 1]^n, \quad \hat{\rho} \geq 0.
\]
Now let \(L_N = \rho h_N\). Clearly c) holds. It is easy to check that
\[
\hat{L}_N(\xi) \geq (1 + |\xi|)^{-n}/C \quad \text{if } |\xi| \leq N - 1,
\]
\[
0 \leq \hat{L}_N(\xi) \leq C|\xi|^{-n} \quad \text{if } \xi \in \mathbb{R}^n.
\]
So if \(\varphi_N\) is the inverse Fourier transform of \((\hat{L}_N)^{1/2}\), then a) and b) hold. Since
\[
|L_N(x) - L_N(y)| \leq |\rho(x) - \rho(y)| |h_N(x)| + \rho(y) |h_N(x) - h_N(y)|,
\]
d) will follow from
\[
|h_N(x)| \leq C \left( \log^+ \left( \frac{1}{|x|} \right) + 1 \right), \tag{8}
\]
and
\[
\left| \frac{\partial}{\partial |x|} h_N(x) \right| \leq \frac{C}{|x|}, \quad |x| \leq 1. \tag{9}
\]
Now
\[
h_N(x) = \int_0^1 \int_{\Sigma_{n-1}} e^{ri \cdot x} d\sigma_{n-1}(\omega) r^{n-1} dr + \int_1^N \int_{\Sigma_{n-1}} e^{ri \cdot x} d\sigma_{n-1}(\omega) \frac{dr}{r},
\]
with the important contribution coming from the second integral. For (8) just use the well-known estimate
\[
\left| \int_{\sum_{n=1}} e^{ix\omega} d\sigma_{n-1}(\omega) \right| \leq \frac{C}{(1 + |x|)(n-1)/2}.
\]
For (9) note that
\[
\int_{\sum_{n=1}} e^{ix\omega} d\sigma_{n-1}(\omega) = \int_0^1 \cos(|x|s)\omega(s) \, ds,
\]
for some \( \omega \in L^1([0, 1]) \). Now
\[
\left| \frac{d}{dt} \int_1^N \int_0^1 \cos(trs) \omega(s) \, ds \, dr \right| = \left| \int_0^1 \int_1^N \sin(trs) \, s \, dr \, \omega(s) \, ds \right|
\leq \int_0^1 \int_s^N \sin(tu) \, du \, \omega(s) \, ds
\leq \frac{C}{|t|}.
\]
Returning to (7) we have, because of our estimate on \( \hat{\mu} \) combined with Lemma 3.a),
\[
\left\| \left( \sum_{\kappa(Q)=j-s} \lambda_Q a_Q \right) * \mu_j \right\|_2^2 = \int_{\mathbb{R}^n} \left| \left( \sum_{\kappa(Q)=j-s} \lambda_Q a_Q \right)(\xi) \right|^2 |\hat{\mu}(2^j\xi)|^2 \, d\xi
\leq C \int_{\mathbb{R}^n} \left| \left( \sum_{\kappa(Q)=j-s} \lambda_Q a_Q \right)(\xi) \right|^2
\cdot \liminf_N \left| \hat{\varphi}_N(2^j\xi) \right|^2 \, d\xi.
\]
Thus, letting \( \varphi_{N,j}(x) = 2^{-nj} \varphi_N(2^{-j}x) \), (7) will follow from the estimates, uniform in \( N \),
\[
\left( \sum_{\kappa(Q)=j-s} \lambda_Q a_Q \right) * \varphi_{N,j} \right\|_2^2 \leq C \alpha(s + 1) \sum_{\kappa(Q)=j-s} \lambda_Q.
\]
So fix $N$, $j$, and $s$ and write $\varphi$ for $\varphi_N$, $\varphi_j$ for $\varphi_{N,j}$. For $q \in \mathcal{R}_{j-s}$, let
\[ A_q = \sum_{\kappa(Q) = j-s} \lambda_Q a_Q, \quad \lambda_q = \sum_{\kappa(Q) = j-s} \lambda_Q. \]

Then
\[
\left\| \left( \sum_{\kappa(Q) = j-s} \lambda_Q a_Q \right) * \varphi_j \right\|_2^2 \leq \sum_{q, q' \in \mathcal{R}_{j-s}} \left| \langle A_q * \varphi_j, A_{q'} * \varphi_j \rangle \right|
\leq \sum_{q'} \sum_{q \subseteq (q')} + \sum_{q'} \sum_{q \cap (q')^* = \emptyset}
= I + II.
\]

The inequality
\[
\| a_Q * \varphi_j \|_2 \leq C 2^{-nj/2}
\]
follows easily from Lemma 3,b) and the well-known estimates
\[
\left| \hat{a}_Q (\xi) \right| \leq C |\xi| \text{diam}(Q),
\]
\[
\| a_Q \|_2^2 \leq \frac{C}{|Q|}.
\]

This leads, via Lemma 2,d), to
\[
I \leq C 2^{-nj} \sum_{q'} \lambda_{q'} \sum_{q \subseteq (q')} \lambda_q
\leq C 2^{-nj} \sum_{q'} \lambda_{q'} \sum_{Q \subseteq (q')^*} \lambda_Q
\leq C 2^{-nj} \sum_{q'} \lambda_{q'} 2^n(j-s+1)
= C \alpha 2^{n(1-s)} \sum_{\kappa(Q) = j-s} \lambda_Q.
\]
To estimate II, begin by fixing \( q, q' \in \mathcal{R}_{j-s} \) with \( q \cap (q')^* = \emptyset \). We write

\[
\langle A_q \ast \varphi_j, A_{q'} \ast \varphi_j \rangle = \int A_q(x)A_{q'} \ast L_j(x) \, dx,
\]

where \( L_j(x) = \varphi_j \ast \tilde{\varphi}_j(x) = 2^{-nj}L(2^{-j}x) \) and so, by Lemma 3.d,

\[
|L_j(x) - L_j(y)| \leq C 2^{-nj} |x - y| \min\{|x|, |y|\}.
\]

Now if \( \kappa(Q) = j - s \), \( Q \subseteq q' \), \( x \in q \), and \( y_0 \in Q \), then

\[
a_Q \ast L_j(x) = \int a_Q(y) (L_j(x - y) - L_j(x - y_0)) \, dy.
\]

Thus

\[
|a_Q \ast L_j(x)| \leq \frac{C 2^{-nj} \operatorname{diam}(Q)}{d(x, Q)} \leq \frac{C 2^{-nj + \sigma(Q)}}{d(x, Q)} \leq \frac{C 2^{-(n-1)j-s}}{d(x, Q)},
\]

since \( \sigma(Q) \leq \sigma(S(Q)) < \kappa(Q) = j - s \) by Lemma 2. Also, if \( a_Q \ast L_j(x) \neq 0 \), then \( d(x, Q) \leq C 2^j \) (since \( L_j \) is supported in \([-2^j, 2^j]^n\)). Thus

\[
|a_Q \ast L_j(x)| \leq \frac{C 2^{-s}}{d(x, Q)^n}.
\]

Now suppose \( x \in q \). If \( Q \subseteq q' \) and \( \kappa(Q) = j - s \), then \( \sigma(S(Q)) < \kappa(Q) = j - s = \sigma(q') \). Since \( S(Q) \cap q' \neq \emptyset, S(Q) \subseteq q' \). Because \( q \cap (q')^* = \emptyset \), we must have \( d(x, S(Q)) \geq 2^{j-s} \). Coupled with \( d(x, S(Q)) \leq d(x, Q) \leq C 2^j \) if \( a_Q \ast L_j(x) \neq 0 \), we estimate, for fixed \( q \in \mathcal{R}_{s-j} \) and \( x \in q \),

\[
\sum_{(q')^* \cap q = \emptyset} |A_{q'} \ast L_j(x)| \leq \sum_{(q')^* \cap q = \emptyset} \sum_{Q \subseteq q', \kappa(Q) = j - s} \sum_{2^{j-s} \leq d(x, S(Q)) \leq C 2^j} \lambda_Q |a_Q \ast L_j(x)|
\]

\[
\leq C \sum_{(q')^* \cap q = \emptyset} \sum_{Q \subseteq q', \kappa(Q) = j - s} \sum_{2^{j-s} \leq d(x, S(Q)) \leq C 2^j} \lambda_Q \frac{2^{-s}}{d(x, Q)^n}
\]

\[
\leq C 2^{-s} \sum_{2^{j-s} \leq d(x, S) \leq C 2^j} \frac{1}{d(x, S)^n} \sum_{Q \subseteq S, \kappa(Q) = j - s} \lambda_Q.
\]
By Lemma 1.a) this last term is dominated by
\[ C \alpha 2^{-s} \sum_{2i-\varepsilon \leq d(x,S) \leq C 2i} \frac{|S|}{d(x,S)^n} \leq C \alpha 2^{-s} \int_{2i-\varepsilon}^{C 2i} \frac{dr}{r} \leq C \alpha 2^{-s}(s + 1). \]
That is, if \( x \in q \), then
\[ \sum_{(q')^* \cap q = \emptyset} |A_{q'} * L_j(x)| \leq C \alpha 2^{-s}(s + 1). \]

Thus, from (13),
\[ \Pi \leq \sum_q \int |A_q(x)| \sum_{(q')^* \cap q = \emptyset} |A_{q'} * L_j(x)| \, dx \]
\[ \leq C \alpha 2^{-s}(s + 1) \sum_q \lambda_q = C \alpha 2^{-s}(s + 1) \sum_{\kappa(Q) = j-s} \lambda_Q. \]

With (11) and (12) this gives (10) and completes the proof of our theorem.

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References.


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