Erratum to: “A categorification of quantum sl(n)”

[Quantum Topol. 1 (2010), 1–92]

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We thank Marco Mackaay, Marko Stošić and Pedro Vaz for pointing out a sign error in Lemma 6.4. It should read as follows:

**Lemma 6.4.** For $i, j \in I$ with $i \neq j$

\[
\Gamma \begin{pmatrix} i & j \\ \lambda + ix & \end{pmatrix} = \begin{cases} H_{k+j} \otimes H_k & \rightarrow H_{+i + j+k+i} \otimes H_{+i + j+k+i}, \\ \xi_j^{\alpha_1} \otimes \xi_i^{\alpha_2} & \rightarrow \xi_i^{\alpha_2} \otimes \xi_j^{\alpha_1}, \\ \end{cases}
\]

\[
\Gamma \begin{pmatrix} \lambda + ix \\ i & j \\ \end{pmatrix} = \begin{cases} H_{+i + j+k+i} \otimes H_{+j+i+k} & \rightarrow H_{k+i} \otimes H_{k+i}, \\ \xi_j^{\alpha_1} \otimes \xi_i^{\alpha_2} & \rightarrow \begin{cases} -\xi_i^{\alpha_2} \otimes \xi_j^{\alpha_1} & \text{if } \begin{array}{c} \xi_i \\
\end{array} \rightarrow \begin{array}{c} \xi_j \\
\end{array}, \\ \xi_i^{\alpha_2} \otimes \xi_j^{\alpha_1} & \text{otherwise.} \\ \end{cases}
\end{cases}
\]

These bimodule maps have degree zero for all $i, j \in I$ and all weights $\lambda$.

In the proof of Lemma 6.4 the sentence “The case when $i = j$ appears in [21] so we will omit this case here” should be removed.

Definition 4.1 in Section 4.2, p. 58–59, should be changed to:

**Definition 4.1.** $\mathcal{U}_{\to}(\mathfrak{sl}_n)$ is an additive $k$-linear 2-category with translation. The 2-category $\mathcal{U}_{\to}(\mathfrak{sl}_n)$ has objects, morphisms, and generating 2-morphisms as described in Definition 3.1, but some of the relations on 2-morphisms are modified.

- The $\mathfrak{sl}_2$ relations and the shift isomorphism relations are the same as before, see equations (3.1)–(3.9).
- Generating 2-morphisms are cyclic with respect to the biadjoint structure, see (3.3) and (3.10), except when $i \cdot j = -1$ we have

\[
\begin{array}{c}
\begin{array}{c}
\xi_i \\
\end{array} \rightarrow \begin{array}{c}
\xi_j \\
\end{array} \\
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{c}
\xi_j \\
\end{array} \rightarrow \begin{array}{c}
\xi_i \\
\end{array} \\
\end{array}
\end{array}
\]
Sideways crossings are defined by the equations

\[
\begin{align*}
\lambda &:= \begin{array}{c}
\includegraphics{sideways_crossing_i}\end{array}, \\
\lambda &:= \begin{array}{c}
\includegraphics{sideways_crossing_j}\end{array}.
\end{align*}
\]

Then the relations (3.13) for \( i \neq j \) become

\[
\begin{align*}
\begin{array}{c}
\includegraphics{sideways_crossing_i} \end{array} \lambda &= \begin{cases} \\
\lambda & \text{if } i \cdot j = 0, \\
(j - i) \begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda - \begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda
\end{array} & \text{if } i \cdot j = -1,
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\includegraphics{sideways_crossing_j} \end{array} \lambda &= \begin{cases} \\
\lambda & \text{if } i \cdot j = 0, \\
(j - i) \begin{array}{c}
\includegraphics{sideways_crossing_j} \lambda & \text{if } i \cdot j = -1.
\end{cases}
\end{align*}
\]

The signed \( R(v) \) relations are:

(a) For \( i \neq j \), the relations

\[
\begin{align*}
\begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda &= \begin{cases} \\
\lambda & \text{if } i \cdot j = 0, \\
(i - j) \left( \begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda - \begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda
\end{array} 
\end{array} \right) & \text{if } i \cdot j = -1.
\end{cases}
\end{align*}
\]

(b) For \( i \neq j \), the relations

\[
\begin{align*}
\begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda &= \begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda, \\
\includegraphics{sideways_crossing_i} \lambda &= \begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda.
\end{array}
\end{array}
\end{align*}
\]

for all \( \lambda \).

(c) Unless \( i = k \) and \( j = i \pm 1 \)

\[
\begin{align*}
\begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda &= \begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda
\end{array}.
\end{align*}
\]

For \( j = i \pm 1 \)

\[
\begin{align*}
\begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda &= (i - j) \left( \begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda - \begin{array}{c}
\includegraphics{sideways_crossing_i} \lambda
\end{array} 
\end{array} \right).
\end{align*}
\]
The rightmost term in the statement of Proposition 6.5 should have a minus sign when \(i \cdot j = -1\). In the statement of Proposition 6.6 when \(i \cdot j = -1\) the first equality should have the sign \((i - j)\) and the second equality should have the sign \((j - i)\).

The 2-category \(\mathcal{U}(\mathfrak{sl}_n)\) described above with modified relations is isomorphic to the 2-category \(\mathcal{U}_\to(\mathfrak{sl}_n)\). The only part of the isomorphism \(\Sigma: \mathcal{U} \to \mathcal{U}_\to\) that needs to be modified is the image of caps and cups. The rescaling of these caps and cups also appears in a work in progress of the second author with Sabin Cautis.

Let \(d_i = (-1)^i\). Then \(\Sigma\) is defined as before except that caps and cups are given as follows:

\[
\Sigma(\begin{array}{c}
\begin{array}{c}
\lambda_i
\end{array}
\end{array}) = \begin{cases}
d_i^{\lambda_i - 1} \begin{array}{c}
\begin{array}{c}
l = 2\ell - 2\ell + 1 - 2\ell - (2\ell + 3)
\end{array}
\end{array} & \text{for } \ell \in 2\mathbb{Z}_{\geq 0}, \\
\begin{array}{c}
l = 2\ell - 2\ell + 1 - (2\ell + 1)
\end{array} & \text{otherwise},
\end{cases}
\]

\[
\Sigma(\begin{array}{c}
\begin{array}{c}
\lambda_i
\end{array}
\end{array}) = \begin{cases}
d_i^{\lambda_i - 1} \begin{array}{c}
\begin{array}{c}
l = 2\ell - 2\ell + 1 - 2\ell - (2\ell + 3)
\end{array}
\end{array} & \text{for } \ell \in 2\mathbb{Z}_{\geq 0}, \\
\begin{array}{c}
l = 2\ell - 2\ell + 1 - (2\ell + 1)
\end{array} & \text{otherwise},
\end{cases}
\]

\[
\Sigma(\begin{array}{c}
\begin{array}{c}
\lambda_i
\end{array}
\end{array}) = \begin{cases}
d_i^{\lambda_i - 1} \begin{array}{c}
\begin{array}{c}
l = 2\ell - 2\ell + 1 - 2\ell - (2\ell + 3)
\end{array}
\end{array} & \text{for } \ell \in 2\mathbb{Z}_{\geq 0}, \\
\begin{array}{c}
l = 2\ell - 2\ell + 1 - (2\ell + 1)
\end{array} & \text{otherwise},
\end{cases}
\]

\[
\Sigma(\begin{array}{c}
\begin{array}{c}
\lambda_i
\end{array}
\end{array}) = \begin{cases}
d_i^{\lambda_i - 1} \begin{array}{c}
\begin{array}{c}
l = 2\ell - 2\ell + 1 - 2\ell - (2\ell + 3)
\end{array}
\end{array} & \text{for } \ell \in 2\mathbb{Z}_{\geq 0}, \\
\begin{array}{c}
l = 2\ell - 2\ell + 1 - (2\ell + 1)
\end{array} & \text{otherwise}.
\end{cases}
\]

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