Erratum: A Parabolic Quasilinear Problem for Linear Growth Functionals

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Abstract

We give the correct proof of Lemma 3.6 of the paper A Parabolic Quasilinear Problem for Linear Growth Functionals (Rev. Mat. Iberoamericana 18 (2002), no. 1, 135-185).

Following [2], let

\begin{equation}
X(\Omega) = \{ z \in L^\infty(\Omega; \mathbb{R}^N) : \text{div}(z) \in L^1(\Omega) \}.
\end{equation}

In [2], the weak trace on \( \partial \Omega \) of the normal component of \( z \in X(\Omega) \) is defined. More precisely, it is proved that there exists a linear operator \( \gamma : X(\Omega) \to L^\infty(\partial \Omega) \) such that

\[ \| \gamma(z) \|_\infty \leq \| z \|_\infty \]

\[ \gamma(z)(x) = z(x) \cdot \nu(x) \quad \text{for all} \quad x \in \partial \Omega \quad \text{if} \quad z \in C^1(\Omega, \mathbb{R}^N). \]

We shall denote \( \gamma(z)(x) \) by \( [z, \nu](x) \).

Assuming that \( \partial \Omega \) is of class \( C^1 \), it is proved in [3] that if \( x_0 \in \partial \Omega \) is a Lebesgue point of the function \( [z, \nu] \), then

\begin{equation}
[z, \nu](x_0) = \lim_{\rho \to 0} \lim_{r \to 0} \frac{1}{2rN-1} \int_{C_{r,\rho}(x_0,\nu(x_0))} z(y) \cdot \nu(x_0) \, dy,
\end{equation}

\( C_{r,\rho}(x, \alpha) \) being the cylinder defined by

\[ C_{r,\rho}(x, \alpha) = \{ y \in \mathbb{R}^N : |(y-x) \cdot \alpha| < r, |(y-x) - [(y-x) \cdot \alpha] \cdot \alpha| < \rho \}. \]

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Our purpose is to give the correct proof of the following Lemma.

**Lemma 1.1 ([1, Lemma 3.6])**

1. Let $u_n \in BV(\Omega) \cap L^2(\Omega)$ and $z \in X(\Omega)$. Suppose that
   \[ a(x, \nabla u_n) \rightharpoonup z \quad \text{weakly}^* \text{ in } L^\infty(\Omega, \mathbb{R}^N) \]
   and
   \[ \text{div}(a(x, \nabla u_n)) \rightharpoonup \text{div} z \quad \text{weakly in } L^2(\Omega). \]
   Then
   \[ [a(x, \nabla u_n), \nu(x)] \rightharpoonup [z, \nu(x)] \quad \text{weakly in } L^2(\partial\Omega) \text{ and} \]
   \[ |z(x) \cdot \nu(x)| \leq f^0(x, \nu(x)) \quad \text{a.e. in } \partial\Omega. \]

2. Let $u_n \in W^{1,2}(\Omega)$. Let $a_n(x, \xi) = a(x, \xi) + \frac{1}{n}\xi$. Suppose that
   \[ \|u_n\|_2 \text{ is bounded in } L^2(\Omega), \]
   \[ \frac{1}{n}|
abla u_n| \to 0 \quad \text{in } L^2(\Omega), \]
   \[ a_n(x, \nabla u_n) \rightharpoonup z \quad \text{weakly in } L^2(\Omega, \mathbb{R}^N) \]
   and
   \[ \text{div}(a_n(x, \nabla u_n)) \rightharpoonup \text{div} z \quad \text{weakly in } L^2(\Omega). \]
   Then
   \[ [a_n(x, \nabla u_n), \nu(x)] \rightharpoonup [z, \nu(x)] \quad \text{weakly in } W^{1/2,2}(\partial\Omega)^* \text{ and} \]
   \[ |[z(x), \nu(x)]| \leq f^0(x, \nu(x)) \quad \text{a.e. in } \partial\Omega. \]

To prove it, let us recall the following result, which corresponds to Lemma 3.7 in [1]. We notice that its proof is independent of Lemma 3.6.
**Lemma 1.2** ([1, Lemma 3.7]) Suppose that any of the assumptions of Lemma 3.6 hold. Moreover we assume that

\[
(1.13) \quad u_n \to u \text{ in } L^2(\Omega) \text{ and } \|u_n\|_{BV} \text{ is bounded},
\]

Then

\[
(1.14) \quad z(x) = a(x, \nabla u(x)) \quad \text{a.e. } x \in \Omega.
\]

**Proof of Lemma 1.1.** Since both proofs are based on similar arguments, we shall only prove ii). For the proof of (1.11) we refer to [1]. Let us prove (1.12). By (1.2), we have

\[
(1.15) \quad [z, \nu](x) = \lim_{\rho \to 0} \lim_{r \to 0} \frac{1}{2rw_{N-1}\rho^{N-1}} \int_{C_{r,\rho}(x,\nu(x))} z(y) \cdot \nu(x) \, dy \quad \mathcal{H}^{N-1}\text{-a.e. on } \partial \Omega.
\]

On the other hand, by assumption \((H_5)\),

\[
a(x, \xi) \cdot \eta \leq f^0(x, \eta)
\]

for all \(\xi, \eta \in \mathbb{R}^N\), and all \(x \in \overline{\Omega}\). Then, by 1.2, we have

\[
(1.16) \quad z(y) \cdot \nu(x) = a(y, \nabla u(y)) \cdot \nu(x) \leq f^0(y, \nu(x)).
\]

Finally, since \(f^0(\cdot, \xi)\) is continuous in \(\overline{\Omega}\) for all \(\xi \in \mathbb{R}^N\), using (1.15), (1.16), we get

\[
|[z, \nu](x)| \leq \lim_{\rho \to 0} \lim_{r \to 0} \frac{1}{2rw_{N-1}\rho^{N-1}} \int_{C_{r,\rho}(x,\nu(x))} |z(y) \cdot \nu(x)| \, dy
\]

\[
\leq \lim_{\rho \to 0} \lim_{r \to 0} \frac{1}{2rw_{N-1}\rho^{N-1}} \int_{C_{r,\rho}(x,\nu(x))} f^0(y, \nu(x)) \, dy = f^0(x, \nu(x)),
\]

\(\mathcal{H}^{N-1}\) – a.e. on \(\partial \Omega\). 

\[\blacksquare\]

**References**


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