Abstract

Let $X = G/H$ be a homogeneous space, $\bar{X} = X \times [0, \infty)$, $\mu$ a doubling measure on $X$ induced by a Haar measure on the group $G$, $\beta$ a positive measure on $\bar{X}$ and $W$ a weight on $X$. Consider the maximal operator given by

$$Mf(x, r) = \sup_{s \geq r} \frac{1}{\mu(B(x, s))} \int_{B(x, s)} |f(y)| \, d\mu(y), \quad (x, r) \in \bar{X}.$$ 

In this paper, we obtain, for each $p, q, 1 < p \leq q < \infty$, a necessary and sufficient condition for the boundedness of the maximal operator $M$ from $L^p(X, Wd\mu)$ to $L^q(\bar{X}, d\beta)$. As an application, we obtain a necessary and sufficient condition for the boundedness of the Poisson integral of functions defined on the unit sphere $S^n$ of the Euclidian space $\mathbb{R}^{n+1}$, from $L^p(S^n, Wd\sigma)$ to $L^q(B, d\nu)$, where $\sigma$ is the Lebesgue measure on $S^n$, $W$ is a weight on $S^n$ and $\nu$ is a positive measure on the unit ball $B$ of $\mathbb{R}^{n+1}$. 