Criteria for extreme points and strongly extreme points of the unit ball in Orlicz spaces with the Orlicz norm are given. These results are applied to a characterization of extreme points of $B(L^{1} + L^\infty)$ which corresponds to the result obtained by R. Grząślewicz and H. Schaefer [Indag. Math. 3 (1992) 173–178] and H. Schaefer [Arch. Math. 58 (1992) 160–163]. Moreover, we show that every extreme point of $B(L^{1} + L^\infty)$ is strongly extreme. We also get criteria for extreme points of $B(L^p \cap L^\infty)$, $1 \leq p < \infty$, using Theorem 1 and for strongly extreme points of $B(L^p \cap L^\infty)$, $1 \leq p < \infty$, applying Theorem 2. Although, criteria for extreme points of $B(L^{1} \cap L^\infty)$ were known [see H. Hudzik, A. Kamińska and M. Mastyło, Arch. Math. 68 (1997) 159–168], we can easily deduce them from our main results and we can extend those results to establish which among extreme points are strongly extreme. The descriptions of the extreme and strongly extreme points of $B(L^p \cap L^\infty)$, $1 < p < \infty$, are original. Moreover, criteria for extreme points and strongly extreme points of the unit ball in the subspace of finite elements of an Orlicz space are deduced on the basis of our main results.