Abstract. The unifying theme of the workshop was the mathematical modeling, analysis, and numerical simulation of materials which involve magnetic and elastic interactions. During the workshop we identified several open problems from the calculus of variations, partial differential equations and modeling which appear to be essential in the understanding of the behavior of magnetoelastic materials.

Mathematics Subject Classification (2010): 35Q74, 35Q35, 49Sxx, 70-XX, 74F15.

Introduction by the Organisers

The mini-workshop *Mathematics of Magnetoelastic Materials*, organized by Carlos J. García-Cervera (Santa Barbara), Martin Kružík (Prague), Chun Liu (University Park), and Anja Schlömerkemper (Würzburg), was attended by 15 participants from the US and Europe. The workshop brought together specialists in mathematical analysis, applied mathematics, numerical computations and engineering. We had 15 extended talks on various aspects of new modeling and mathematical approaches to magnetoelastic materials. Finally, we had a lively closing discussion on open problems. The atmosphere of the mini-workshop was stimulating and very collaborative. During every talk, several questions were posed and interesting problems were pointed out.

Magnetoelectricity describes the mechanical behavior of solids under magnetic effects. The magnetoelastic coupling is based on the presence of small magnetic domains in the material. In the absence of an external magnetic field, these magnetic domains are randomly oriented but when exposed to a field they become...
aligned along the field and their rotations induce a deformation of the specimen. As the intensity of the magnetic field increases, more and more magnetic domains orientate themselves so that their principal axes of anisotropy are collinear with the magnetic field in each region and finally saturation is reached. The mathematical modeling of magnetoelasticity is a vibrant area of research, triggered by the interest on ferromagnetic shape-memory alloys, magnetorheological elastomers and magnetic fluids. Mathematical tools include weak convergence methods, lower semicontinuity, compactness, multiscale methods, homogenization, and various approximations.

The main open problems that we worked out during the workshop are:

- What is the role and importance of the saturation constraint for the magnetization with respect to modeling and analysis?
- Resolve the discrepancy between the modeling of different physical effects that occur on different length scales, as e.g., exchange energies and rate-independent dissipation mechanisms.
- What are reasonable models for dissipation? Can one derive them from microscopic considerations?
- Show (non-)existence of minimizers of energy functionals arising in magnetoelasticity.
- Can one read off hysteresis from material parameters? What is the role of nucleation?
- What is the effective static and dynamical behavior of composite materials with microscopic structure and magnetic interactions?

These are important problems which will help to construct more accurate mathematical models for magnetoelasticity. We are convinced that they will also lead to new mathematical developments.

Generally speaking, the participants felt that the mini-workshop format with fifteen researchers was particularly successful in promoting discussions and new interactions. The organizers thank the Institute’s staff for having provided an inspiring and comfortable environment for the participants.

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