Abstract. The PBW structures play a very important role in the Lie theory and in the theory of algebraic groups. The importance is due to the huge number of possible applications. The main goal of the workshop was to bring together experts and young researchers working in the certain areas in which PBW structures naturally appear. The interaction between the participants allowed to find new viewpoints on the classical mathematical structures and to launch the study of new directions in geometric, algebraic and combinatorial Lie theory.

Mathematics Subject Classification (2010): 17B10, 17B35, 20G05, 14M15, 14M25, 52B20, 05E05, 05E10.

Introduction by the Organisers

The mini-workshop *PBW Structures in Representation Theory*, organised by Evgeny Feigin (Moscow), Ghislain Fourier (Glasgow) and Martina Lanini (Edinburgh) took place February 28th–March 5, 2016. It was attended by 17 participants, including a consistent number of young researchers, from France, Germany, Italy, Russia, Spain, the UK and the US.

The first two days of the mini-workshop were dedicated to four two-hour introductory lecture series, whose aim was to provide all participants with some common background. They recalled the state of art, explained open and interesting conjectures, and gave possible directions for future projects and interactions. Within the week, we also had 7 research talks, most of them given by the younger
participants. All presentations were quite informal, with a very active participation of the audience, strong interactions with the speaker, and several questions and answers during the talks, which served as a starting point for the discussion sessions. We have the evidence that the workshop was properly sized and quite focused as in all talks, almost all participants were directly involved in the interactions. We have had daily discussion sessions which played a central role in the miniworkshop. These informal sessions were of two types: either a participant was explaining a problem he/she is working on, or a group of participants were working together on an existing or new project.

The topics discussed within the workshop were centred around the PBW (Poincaré-Birkhoff-Witt) theorem and its appearances and applications in different areas of mathematics, such as representation theory, algebraic geometry, and combinatorics. The central idea is to consider an algebraic object, e.g. a simple complex Lie algebra, and apply the PBW Theorem to produce a filtration on it. This gives a machinery to generate new mathematical objects starting from a given one by applying certain algebra or group of operators, it provides a powerful link between Lie theory and commutative algebra. Such a strategy has been successfully applied in (algebraic, geometric and combinatorial) Lie theory and proved to be very powerful both in the theoretical questions and in applications.

The goal of the mini-workshop was to further study this sort of phenomena and, in particular, investigate applications in new fields by bringing together researchers with different backgrounds. Indeed, the range of topics discussed within the workshop was broad: Newton-Okounkov bodies, toric degenerations, quiver Grassmannians, affine and finite-dimensional Grassmannians and flag varieties, representation theory of Kac-Moody Lie algebra, theory of Macdonald polynomials (symmetric and nonsymmetric). The interdisciplinary of the event was further fostered by interesting exchanges with several researchers working on toric degenerations and Newton-Okounkov bodies, who were attending a parallel MFO mini-workshop (“Arrangements of Subvarieties, and their Applications in Algebraic Geometry”).

Several new projects were initiated because of the workshop. The exchanges between experts in representation theory and experts of Newton-Okounkov bodies led to very promising new collaborations. The concentration of researchers coming from different areas (cluster algebras, Lie algebras, tropical geometry) but working on the common ground of toric degenerations has given a much needed impact on the project of relating all these various points of view. Further, some of the world leading researchers in the representation theory of current algebras were part of the workshop, and helped to investigate further on the relation between this representation theory, the PBW filtration and Macdonald polynomials.

These were just three of several discussed topics and initiated collaborations, we are looking forward to seeing the results emerging from this workshop in the near future.

Summarizing, the MFO mini-workshop PBW Structures in Representation Theory was very successful and succeeded all high expectations of the organisers.
Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”.
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