Interplay of Analysis and Probability in Applied Mathematics

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26 July – 1 August 2015

Abstract. This workshop brought together analysts and probabilists working on problems at some of the many interfaces of these two fields. Most of the problems discussed during the meeting have their origin in physics or chemistry. The workshop was grouped around the four themes (1) condensation in random structures, (2) disordered systems, (3) discrete-to-continuum transitions, (4) atomistic and molecular systems.

Mathematics Subject Classification (2010): 60xx, 70xx, 74xx, 82xx.

Introduction by the Organisers

Stochastic effects play an increasingly important rôle in the mathematical modeling of many physical systems. On the mathematical side, this creates the need to combine analytic and stochastic techniques. In addition, a number of challenging and important problems only become tractable when methods from analysis and probability are combined.

In recent decades, more and more researchers in both of these areas became aware of the increasing need to interact with and learn from each other, and many of them took up serious attempts to contribute. The organisers feel that this development is very promising, is likely to bear many qualitatively new results and research directions in future, and should therefore be strongly supported. This is why already in 2005, when the need for collaboration between analysis and probability was less noticed than it is now, the organisers (in a different composition) applied for the first Oberwolfach workshop focused on this aspect. This
and forthcoming applications lead to the organisation of two such workshops in 2008 and 2012. We like to think that these workshops helped significantly to promote collaboration between the two fields. Their positive effects became especially clear during the present workshop, the third one on the interplay of analysis and stochastics in Oberwolfach. It had a very lively and inspiring atmosphere and clearly showed that in the meantime the two communities have got accustomed with each other, and that there is nowadays a lot of high-level exchange between them. Very helpful in this respect was the partial substitution of some of the proposers for the present workshop, which brought in new aspects and research directions.

The aim of this workshop was to bring together analysts and probabilists to foster an exchange of expertise between statistical physics, analysis and probability. There were a number of themes where connections between the different disciplines arise naturally. An example is stochastic averaging, where analytic methods ($\Gamma$-convergence and Young measures) are used to describe the effective macroscopic behaviour of systems with micro- or mesoscopic stochasticity (Ci-calese and Sandier). These approaches were complemented by quantitative estimates for stochastic homogenisation (Neukamm). The treatment of many particle quantum mechanics using statistical mechanics is another example (Sütö, Lukkarinen).

A further natural link between analysis and probability is provided by diffusion processes. On this workshop, various aspects were covered, both on the level of Brownian motions (Berger) and on the level of the Fokker-Planck equations and their numerical treatment (Pavliotis). The treatment of viscid flows via an optimal transport problem involving the Nelson derivative broadened the perspective to fluid mechanics (Leonard). The metric structure given by the Wasserstein distance also proved essential in the convergence result of Kac's model of elastic collisions to the homogeneous Boltzmann equation (Norris).

Large deviations underpinned a number of talks, including compactness questions for the mean-field polaron model (Mukherjee), rate functionals for chemical reactions (Renger), and a new passage from particles to hydrodynamic limits (Fathi). The latter approach also relies on $\Gamma$-converging, thus bridging methodologically to some of the homogenisation approaches discussed above. Two recent and technically quite sophisticated Wulff shape results for an important model from statistical mechanics (bond percolation on the square lattice) were discussed (Biskup). Applications of the large deviation theory by Freidlin and Wentzell to rare events for turbulent atmosphere jets were presented (Bouchet). In many, if not all, of these examples, the structure of the variational formula and the characteristic equations for the zeros of the rate function, particularly non-gradient terms, play a central rôle.

A different approach to study metastability and transition probabilities relies on capacity estimates (Schlichting). This is one of several examples where similar problems were studied by different parts of the community, and the intention of the workshop was to foster the exchange. Further work on metastability, in the context
of Kawasaki dynamics in Ising spin systems with Kac potentials, was presented (de Masi).

A number of talks were devoted to the aim of understanding the emergence of structures in multi-agent systems. Examples are condensation in stochastic networks (Dereich, Mörters), crystallisation in the low-temperature limit (Jansen), emergence of phases in complex networks (Radin), ground state analysis of interaction energies appearing in particle dynamics such as flock formation (Cañizo).

Combinatorial problems were behind a number of ingredients of arguments, with entropy being only the most obvious example. These problems also were discussed as challenges in their own right, often with application from physics in mind (Aurzada, Gneden, Zeindler). The formulation often was in term of fundamental limit laws of probability distributions, like central limit theorems and point process convergence; another example was such a convergence result for the extremes of bivariate point processes, derived by means of Stein’s method (Cipriani).

There were a number of talks by young participants (Duhart, Einav, Flegel, Mönch, Stamatakis, Taggi, Williams); the quality of their results and talks was impressive. The stimulating environment of Oberwolfach helped foster a number collaborations during the workshop, and new results were obtained during the meeting.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”. Moreover, the MFO and the workshop organizers would like to thank the Simons Foundation for supporting Marek Biskup in the “Simons Visiting Professors” program at the MFO.
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