Abstract. Coideal subalgebras of quantized enveloping algebras appear naturally if one considers quantum group analogs of Lie subalgebras. Examples appear in the theory of quantum integrable systems with boundary and in harmonic analysis on quantum group analogs of Riemannian symmetric spaces. Recently, much progress has been made to develop a deeper representation theoretic understanding of these examples. On the other hand, coideal subalgebras play a fundamental role in the theory of Nichols algebras. The workshop aimed to discuss these theories in view of the recent developments.

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Introduction by the Organisers

Quantum groups are well-known objects in representation theory, in the theory of integrable systems, and in the theory of Hopf algebras. Quantum group analogs of homogeneous spaces were much slower to develop, mainly because the immediate definition ((co)invariants with respect to a Hopf subalgebra) turned out to be too rigid to allow for interesting examples. By now, however, there is ample evidence that one-sided coideal subalgebras of quantum groups provide a suitable concept of quantum homogeneous spaces.

The relation between quantum integrable systems with boundary and comodule algebras for quantum groups was apparent early on. About 15 years ago, explicit examples of coideal subalgebras of quantum groups in Drinfeld-Jimbo realization appeared in the investigation of quantum integrable systems with boundary. These coideal subalgebras provided a tool to construct solutions of the so called reflection
equation, an integrability condition for systems with boundary. In the 90s, a theory of quantum group analogs of symmetric spaces was developed by M. Noumi et al. and independently by G. Letzter. The aim of their program was to provide new interpretations of Macdonald-Koornwinder polynomials as zonal spherical functions on quantum symmetric spaces. The construction of quantum symmetric pairs has recently been extended to involutive automorphisms of symmetrizable Kac-Moody algebras by S. Kolb. It includes the examples obtained from integrable systems.

Very recently, H. Bao & W. Wang and M. Ehrig & C. Stroppel indicated that much of modern representation theory for quantum groups (Schur-Jimbo duality, canonical bases, Kazhdan-Lusztig theory, categorification) extends to quantum symmetric pairs. From the algebraic side, coideal subalgebras of quantum groups have been classified within the wider context of Nichols algebras, a development which provides ample new technology.

The workshop brought together experts on coideal subalgebras from different backgrounds (integrable systems, special functions, quantum symmetric pairs, representation theory, Nichols algebras). The aim was to get to know each others perspective, to analyze the present state of the art, and to pursue avenues of cross-fertilization.

The workshop started out with three mini lecture series. N. Reshetikhin gave an introduction to the physics origins of the reflection equation. Starting from the 6-vertex model in statistical mechanics he introduced the boundary $q$-Knizhnik-Zamolodchikov equations as the consistency conditions for correlation functions with reflecting boundary. C. Stroppel gave an overview of categorification of quantum groups in view of recent developments for coideal subalgebras. H.-J. Schneider delivered a crash course on Nichols algebras. He explained how right coideal subalgebras form a potent tool within this general theory and outlined their classification in terms of the Weyl groupoid.

The talks by the remaining 13 participants gave insight into different aspects of coideal subalgebras of quantum groups. The recent developments around Bao & Wang’s program of canonical bases and Ehrig & Stroppel’s categorification for quantum symmetric pairs generated a lot of enthusiasm. One central ingredient here is a new bar involution for these coideal subalgebras. This has already been very fruitful in the construction of a universal $K$-matrix in the finite setting, see M. Balagović’s talk. Several talks highlighted the recent important developments regarding the role of coideal subalgebras in integrable systems with boundaries.

We left the workshop with the impression that this research area has made a big leap forward and that further developments are to be expected soon.

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Mini-Workshop: Coideal Subalgebras of Quantum Groups

Table of Contents

Martina Balagovic (joint with Stefan Kolb)
   Universal solutions of the reflection equation .......................... 537
Huanchen Bao
   Canonical bases arising from quantum symmetric pairs ................. 539
Pascal Baseilhac
   The algebra $A_q$, $q$—Onsager algebras and coideal subalgebras: two open problems ................................................. 541
Michael Ehrig (joint with Catharina Stroppel)
   Coideal subalgebras and three dualities .................................. 544
Erik Koelink (joint with Noud Aldenhoven, Pablo Román)
   Matrix-valued orthogonal polynomials associated to an explicit quantum symmetric pair ................................................. 546
Stefan Kolb
   Coideal subalgebras of quantum groups ................................... 548
Simon D. Lentner
   Different types of quantum groups and the Frobenius homomorphism ... 550
Gail Letzter
   An Overview of Quantum Symmetric Pairs ................................ 552
Vidas Regelskis (joint with Bart Vlaar)
   $K$-matrices: from twisted Yangians to quantum loop algebras ........ 554
Nicolai Reshetikhin
   Boundary reflection equation (physics origins) .......................... 558
Hans-Jürgen Schneider (joint with István Heckenberger)
   Nichols algebras and their right coideal subalgebras .................... 558
Gus Schrader
   Integrable systems from the classical reflection equation ............. 560
Jasper V. Stokman
   Boundary quantum Knizhnik-Zamolodchikov equations ................ 561
Catharina Stroppel
   Quantum groups and coideals and categorification ..................... 563
Bart Vlaar (joint with Nicolai Reshetikhin and Jasper V. Stokman)
   Boundary quantum $KZ$ equations — integral solutions ............... 564