Abstract. The 2015 Oberwolfach meeting “Geometric and Algebraic Combinatorics” was organized by Gil Kalai (Jerusalem), Isabella Novik (Seattle), Francisco Santos (Santander), and Volkmar Welker (Marburg). It covered a wide variety of aspects of Discrete Geometry, Algebraic Combinatorics with geometric flavor, and Topological Combinatorics. Some of the highlights of the conference included (1) counterexamples to the topological Tverberg conjecture, and (2) the latest results around the Heron-Rota-Welsh conjecture.

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Introduction by the Organisers

The 2015 Oberwolfach meeting “Geometric and Algebraic Combinatorics” was organized by Gil Kalai (Hebrew University, Jerusalem), Isabella Novik (University of Washington, Seattle), Francisco Santos (University of Cantabria, Santander), and Volkmar Welker (Philipps-Universität Marburg, Marburg).

The conference consisted of six one-hour lectures on outstanding recent developments in the field plus about twenty five shorter talks, ranging from 30 to 45 minutes. On Thursday evening there was a problem session and on Wednesday evening two informal working sessions organized by the participants: one on the face numbers of spheres and manifolds and another one on the topological methods in combinatorics. There were of course many other small group discussions. All together it was a very productive and enjoyable week.
The conference treated a broad spectrum of topics from Topological Combinatorics (such as very recent counterexamples to the topological Tverberg conjecture — this was one of the extremely exciting highlights of the conference — more on it below), Tropical Geometry (such as enumeration of curves, valu ated matroids, tropical laplacians, etc.), Geometric Combinatorics (triangulated manifolds, random simplicial complexes, etc.), Discrete Geometry (diameters of polytopes, lattice points in rational polytopes, complexity, etc.), and Combinatorial Algebra (Stanley’s partitioning conjecture, Gröbner bases of algebras associated with matroids, etc.). It is impossible to summarize in a one-page report the richness and depth of the work and the presentations. Instead we will concentrate here on some of the highlights.

The very first lecture on Monday was given by Karim Adiprasito and was devoted to the c-arrangements. An old conjecture (from late sixties) of Heron, Rota, and Welsh posits that a certain sequence of numbers associated with a matroid, called the Whitney numbers of the first kind, is log-concave. A few years ago this conjecture was solved by Huh and Katz for the case of realizable matroids utilizing the heavy machinery of algebraic geometry. In his talk, Adiprasito announced a far reaching generalization of the Huh–Katz’s result (joint with Raman Sanyal) asserting that this conjecture holds for general linear c-arrangements. Perhaps the most stunning part of this presentation was the sketch of the proof: it relied on the Alexandrov-Fenchel inequality and on probabilistic methods such as the measure concentration techniques!

Tuesday’s morning session was devoted to linear programming/optimization. One of the highlights of this session was a surprising connection between tropical geometry and optimization discussed in Michael Joswig’s talk. Michael and his coauthors exploited this connection to disprove a continuous analog of the Hirsch conjecture proposed by Deza, Terlaky and Zinchenko.

Spectacular recent developments in topological combinatorics were presented in Tuesday afternoon’s session. Specifically, Uli Wagner reported on his (joint with Isaac Mabillard) recent major theory that extends fundamental theorems from classical obstruction theory for embeddability to an obstruction theory for r-fold intersection of disjoint faces in maps from simplicial complexes to Euclidean spaces. This presentation was followed by Florian Frick’s talk who building on Mabillard–Wagner’s theory and on a result by Özbaydin, presented his 3-page preprint refuting the topological Tverberg conjecture! This was a truly fascinating sequence of talks!

On Thursday, Roy Meshulam talked about the recent results on high-dimensional expansion. His lecture was followed by Nati Linial’s account of the recent progress on random simplicial complexes.

Several talks were devoted to the recent progress on the face numbers of simplicial complexes. Afshin Goodarzi discussed his joint work with Karim Adiprasito and Anders Björner in which they characterized face numbers of sequentially Cohen-Macaulay complexes, thus providing an impressive generalization of the classical Macaulay-Stanley theorem to the nonpure case. Kalle Karu presented his
recent work on the $cd$-index; specifically, he proved that the Murai–Nevo conjecture on the $cd$-index of Gorenstein posets holds for the case of simplicial spheres. Satoshi Murai announced his (joint with Martina Juhnke-Kubitzke) very recent proof of the balanced generalized lower bound conjecture for simplicial polytopes posed by Klee and Novik. Ed Swartz closed the conference with the talk titled “What’s next?” in which he surveyed recent developments on the face numbers of triangulated manifolds along with several old and new open problems, among them a recent conjecture by Bagchi and Datta. This conjecture posits bounds on the face numbers of triangulated manifolds in terms of the $\mu$-numbers – certain invariants related to Morse theory, topological tightness, and commutative algebra.

It bears repeating that quite a few breakthrough results were announced and presented for the first time during the conference. These include the counterexamples to the topological Tverberg conjecture, the proof of the generalized lower bound conjecture for balanced polytopes, etc. In fact, several math arXiv preprints that are less than two months old and were presented at the workshop (and three of them were actually submitted to the arXiv during the workshop!), see


Last but not least, there was a lively and incredible problem session: a large number of the problems/questions raised were answered on spot.

The collection of abstracts below presents an overview of the official program of the conference. It does not cover all the additional smaller presentations, group discussions and blackboard meetings, nor the lively interactions that occurred during the week. However, it does convey the manifold connections between the themes of the conference, refinements of well-established bridges, completely new links between seemingly distant themes, problems, methods, and theories, as well as demonstrates substantial progress on older problems. In short, it shows that the area is very much alive!

We are extremely grateful to the Oberwolfach institute, its director and to all of its staff for providing a perfect setting for an inspiring, intensive week of “Geometric and Algebraic Combinatorics”.

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Gil Kalai, Isabella Novik, Francisco Santos, Volkmar Welker
Jerusalem/Seattle/Santander/Marburg, March 2015
## Workshop: Geometric and Algebraic Combinatorics

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