Flat Surfaces and Dynamics on Moduli Space

Organised by
Howard Masur, Chicago
Martin Möller, Frankfurt
Anton Zorich, Paris

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Abstract. Dynamics of the Teichmüller geodesic flow on the moduli space of curves and asymptotic monodromy of the Hodge bundle along this flow have numerous applications to dynamics and geometry of measured foliations, to billiards in polygons, to interval exchange transformations, and to geometry of flat surfaces.

Mathematics Subject Classification (2010): 30F, 32G, 37D, 37E, 37H, 57M.

Introduction by the Organisers

The workshop Flat surfaces and Dynamics on moduli space brought together over 53 participants, many of them very young and active. (We did not take statistics, but we believe the average age of the participants to be below 35.) This area is currently extremely active. There were a number of major results presented at the conference that were obtained less than five months since the previous conference on a similar subject at ICERM (Providence). This included the example of a new $GL_2(\mathbb{R})$-invariant suborbifold of an absolutely mysterious origin found by Mirzakhani and Wright, and a strategy for proving the existence (and, actually, genericity) of infinite cyclic Veech groups presented by Hamenstädt.

The background and topics of this workshop included dynamical systems, geometric topology, and algebraic geometry, as reflected in the following summary of some of the main lines of research.

The description of orbit closures of various flows on the moduli spaces is a guiding problem in this field. Major progress has been made in this direction recently. The talks of Aulicino, Bainbridge, Filip, Mohammadi, Nguyen and Wright presented progress on the $SL_2(\mathbb{R})$-orbit closures. Smillie and Weiss reported on the
corresponding problem for the horocycle flow. Counting problems on billiards and
the connection to intersection theory on moduli spaces were addressed by Athreya,
Chen, Goujard, and Zograf.

The geometry of individual flat surfaces is also an active topic, both from the
viewpoint of dynamics (as reported by Chaika, Lelièvre) and group theory (see
the talks on Veech groups by Lehnert, Weitze-Schmithüsen). The dynamics of
translation surfaces of infinite type is a new branch in this field and has developed
recently. Progress in this direction was reported on by Hooper, Treviño, and
Valdez.

Ties with algebraic geometry varying from Shimura curves to p-adic origamis
were discussed by Grushevsky, Herrlich, Kappes, Mondello, and Mukamel. The
talks of Delecroix, Eskin, Fei Yu, Hubert were devoted to various aspects of the
study of the Lyapunov exponents of the Hodge bundle and dynamical Hodge
decomposition: from the relation to the Harder–Narasimhan filtration to applications
to windtree billiards.

Talks in the conference unified the dynamical counterparts of such subjects as
multidimensional diophantine approximations and random walks on groups as in
the talks of Bufetov, Cheung, Gouëzel, to the geometric ones such as the counting
of closed geodesics and study of nonclosed geodesics in the moduli space, and
evaluating the lengths of the corresponding systols. These latter subjects were
discussed in the talks of Boissy, Hamenstädt, and Lenzhen.

There was a broad variety of techniques in the presentations including a beau-
tiful artistic movie of Davis using dancing to illustrate the cutting sequences of
the flow in the double pentagon and multi-zooming software used by Athreya.

We have extraordinarily strong women colleagues in our area. Eight of them
participated in the conference; the results of those who were unable to come, such
as Maryam Mirzakhani, were presented by their collaborators.

The participants intensely discussed mathematics and worked between the talks
and in the evenings. We expect to see many new results to soon emerge from these
discussions.
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