Abstract. Cluster algebras are a class of commutative algebras introduced by Fomin and Zelevinsky in 2000. Their original purpose was to obtain a combinatorial approach to Lusztig’s dual canonical bases of quantum groups and to total positivity. Since then numerous connections between other areas of mathematics have been discovered. The aim of this workshop was to further strengthen these connections and to develop interactions.

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Introduction by the Organisers

The workshop Cluster Algebras and Related Topics, organised by Bernhard Keller (Paris 7), Bernard Leclerc (Caen) and Jan Schröer (Bonn) was attended by 52 participants coming from various different areas of mathematics. There were 23 one hour lectures given at the meeting allowing ample time for questions and discussions.

Cluster algebras were introduced by Fomin and Zelevinsky in 2000. They are by definition a class of commutative algebras with an inductively constructed set of algebra generators called cluster variables. These are grouped together in finite overlapping subsets (called clusters) of a given size. Starting with an initial cluster the other clusters and cluster variables are obtained via a combinatorially defined process called mutation.

The positivity conjecture (saying that all cluster variables are positive Laurent polynomials in any given cluster) was solved only very recently by Schiffler and Lee. Schiffler presented their proof in the opening talk of the conference.
The search for bases of cluster algebras with favourable properties (such as positivity) is still wide open. However substantial progress has been made for some classes of cluster algebras. Thurston presented in his talk a construction of a positive basis for skein algebras, which are intimately related to cluster algebras arising from triangulations of marked surfaces. For arbitrary skew-symmetric cluster algebras Plamondon presented his joint work with Cerulli, Keller and Labardini showing that all cluster monomials are linearly independent. The proof uses a categorification of cluster algebras via generalized cluster categories.

Geiß explained in his talk that for most mutation-finite quivers there exists up to equivalence only one non-degenerate potential. This implies that for such quivers there is essentially just one generalized cluster category. Labardini (joint work with Zelevinsky) presented a new approach via representations of species in an attempt to generalize Derksen, Weyman and Zelevinsky’s additive categorification via Jacobian algebras from skew-symmetric to skew-symmetrizable cluster algebras. Iyama (joint work with Reiten and Adachi) presented a report on \( \tau \)-tilting, a new framework for categorifying cluster algebras.

An important question is which algebras carry natural cluster algebra structures. Yakimov (joint work with Goodearl) presented a new ring theoretical approach for the construction of quantum cluster algebra structures on numerous quantum coordinate algebras arising in Lie theory. A different link between cluster algebras and Lie theory was the content of Gekhtman’s talk on Poisson-Lie groups and cluster algebras.

The homogeneous coordinate rings of Grassmannians carry a natural cluster algebra structure by work of Scott. King presented a new categorification of these Grassmann cluster algebras, in terms of Cohen-Macaulay modules over a twisted group ring. The interaction between Grassmann cluster algebras, the classical combinatorics of Grassmannians and dimer models have been the leading theme of the talks of Baur, Muller, Musiker and L. Williams.

The link between complex integrable systems and cluster algebras was the topic of Fock’s talk and of Soibelman’s report on his groundbreaking work with Kontsevich. The strong connections between mathematical physics and cluster algebras were also discussed in the talks by Neitzke and H. Williams and Di Francesco.

Other interesting topics related with cluster algebras have been presented: cohomology of cluster varieties (Chapoton), complex volume of knots (Inoue), generalized friezes and cluster categories (Jorgensen, joint work with Holm), noncommutative cluster algebras (Retakh, joint work with Berenstein).

Last not least the week was filled with many informal discussions. The workshop provided a perfect atmosphere for exchanging ideas and strengthen interactions. It is our pleasure to thank the administration and the staff of the Oberwolfach Institute for their support and hospitality.
# Workshop: Cluster Algebras and Related Topics

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