Abstract. The notion of soficity for a group is a weak type of finite approximation property that simultaneously generalizes both amenability and residual finiteness. In 2008 L. Bowen discovered how it can be used to significantly broaden the scope of the classical theory of dynamical entropy beyond the setting of amenable acting groups. This Arbeitsgemeinschaft aimed to provide a comprehensive picture of the subject of sofic entropy as it has developed over the last five years.

Mathematics Subject Classification (2010): 37A35.

Introduction by the Organisers

The Arbeitsgemeinschaft Sofic Entropy was organized by Lewis Bowen (Austin) and David Kerr (College Station) and held from October 6 to 11, 2013. There were more than 40 participants, a large proportion of which were graduate students and postdocs. Many participants came with expertise in closely related subjects like infinite group theory, operator algebras, and graph theory, while others represented areas such as number theory and coarse geometry, making for a lively mixture of backgrounds and interests. There was a total of 18 lectures, each one hour in length.

The meeting aimed to address the main concepts and results in the theory of sofic entropy as it has developed starting from the seminal work of Bowen five years ago. The concept of entropy was introduced into ergodic theory by Kolmogorov in the late 1950s with motivation from Shannon’s information theory. An analogous theory of topological entropy was initiated by Adler, Konheim, and McAndrew in the early 1960s, and the two entropies, measure and topological, are related by a variational principle. These classical approaches to dynamical entropy
involve averaging across partial orbits and thus are ultimately suited to actions of amenable groups, for which much of the basic theory was developed by Ornstein and Weiss. Ornstein and Weiss showed in particular that entropy is a complete invariant for Bernoulli actions of a countably infinite amenable group, extending a celebrated result of Ornstein for single Bernoulli shifts.

In the broader realm of sofic acting groups, Bowen showed that one could produce an entropy invariant by externalizing the averaging in the classical amenable definition of entropy to a finite set on which the group acts in an approximate sense, according to the definition of soficity. This led to an entropy classification for Bernoulli actions of all countably infinite nontorsion sofic groups. A more general definition of measure entropy and a corresponding notion of sofic topological entropy were subsequently introduced by Kerr and Li, who also established a variational principle relating the two. Sofic entropy has been applied for example to the study of algebraic actions and questions surrounding the Fuglede-Kadison determinant in group von Neumann algebras, and it has also inspired the development of sofic dimension for groups and equivalence relations. These topics were all covered in the lectures, as well as Gottschalk’s surjunctivity problem, the $f$-invariant, sofic mean dimension, the computation of sofic entropy for algebraic actions, combinatorial independence, Li-Yorke chaos, and entropy in the framework of groupoids.
Arbeitsgemeinschaft: Sofic Entropy

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