Lattice Differential Equations

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Abstract. The workshop focused on recent advances in the analysis of lattice differential equations such as discrete Klein-Gordon and nonlinear Schrödinger equations as well as the Fermi-Pasta-Ulam lattice. Lattice differential equations play an important role in emergent directions of modern science. These equations are fascinating subjects for mathematicians because they exhibit phenomena, which are not encountered in classical partial differential equations, on one hand, but they may present toy problems for understanding more complicated Hamiltonian differential equations, on the other hand.

Mathematics Subject Classification (2010): 34K31, 34A33.

Introduction by the Organisers

The workshop was well attended with 24 participants with broad geographic representation from all continents. This workshop was a nice blend of researchers with various backgrounds interested in lattice differential equations such as discrete Klein-Gordon and nonlinear Schrödinger equations as well as the Fermi-Pasta-Ulam lattice. Nonlinear differential-difference equations for the dynamics of localized excitations in lattices have been recently studied in a number of physical contexts. The novelty and significant differences of these models from the classical nonlinear partial differential equations make the topic of lattice dynamics fascinating for researchers. New phenomena can arise in lattice differential equations such as bifurcation of large-amplitude breathers from infinity, multiple resonances with linear waves in polyatomic models, and propagation failure for non-smooth nonlinear potentials. At the same time, mathematical analysis of
differential-difference equations may be simplified in some problems because the difference operators are bounded, strong solutions of initial-value problems exist globally in time, and the system of differential equations can be uncoupled in the anti-continuum limit. Recent applications of the lattice differential equations are important for inter-disciplinary studies between mathematics, physics, biology, and electrical engineering. These applications call mathematicians to contribute to the analysis of lattice differential equations. Among other applications, we singled out the following particular topics of increasing interest.

- Fermi-Pasta-Ulam problems with non-analytic potentials are considered in the context of granular crystals, where particles interact according to Hertzian forces. Understanding and controlling localization in granular crystals may lead to new engineering devices related to energy filters.
- Discrete Klein-Gordon lattices are used in the Peyrard-Bishop model of the base pairs in DNA. The Morse potential is confining for negative displacements and bounded for positive displacements. Intrinsic localized modes bifurcate with large amplitudes in these models and may be relevant for the analysis of global dynamics of base pairs in DNA.
- Nanophotonics, engineering of photo-refractive crystals, and trapping of atomic Bose-Einstein condensates in optical lattices, all rely on modeling of the discrete nonlinear Schrödinger equation in the space of one, two, and three dimensions. Recent works on discrete vortices and Gibbs measure for phase transitions to solitons have stimulated further studies of dynamics of localization at large energies in these models.

Mathematical studies of lattice differential equations are developed by using various popular approaches, like bifurcation theory, dynamical systems methods, applied harmonic analysis, perturbation theory, numerical simulations, KAM theory, and symplectic geometry. The intensity and extent of recent developments and the increasing amount of interesting problems arising from applications render especially timely a meeting that will bring specialists in the relevant areas together. The workshop focused on aspects related to the dynamics of the above lattice differential equations and on connections with relevant applications. In particular, the workshop covered the following concentration areas.

1. Resonances in lattice equations and macroscopic analysis;
2. Spatially localized oscillations in nonlinear lattices, applications to DNA dynamics;
3. Travelling waves in lattices with limited smoothness, applications to granular crystals;
4. Orbital and asymptotic stability of solitons in lattices; and
5. Vortices in multi-dimensional lattices and justification of variational approximations.
# Workshop: Lattice Differential Equations

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