Abstract. The workshop ‘Group Theory, Measure, and Asymptotic Invariants’ organized by Miklos Abertz (Budapest), Damien Gaboriau (Lyon) and Andreas Thom (Leipzig) was held 18 - 24 August 2013. The event was a continuation of the previous Oberwolfach workshop ‘Actions and Invariants of Residually Finite Groups: Asymptotic Methods’ organized by Miklos Abertz (Budapest), Damien Gaboriau (Lyon) and Fritz Grunewald (Dusseldorf) that was held September 5 - September 11, 2010. Fritz Grunewald passed away in March 2010 and Andreas Thom joined the organizing team.

The workshop aimed to study finitely generated groups and group actions using ergodic and measure theoretic methods, incorporating asymptotic invariants, such as $\ell^2$-invariants, the rank gradient, cost, torsion growth, entropy-type invariants and invariants coming from random walks and percolation theory.

The participant body came from a wide range of areas: finite and infinite group theory, geometry, ergodic theory, graph theory, topology, probability theory, representation theory, von Neumann algebras and $\ell^2$-theory. The participants typically did not speak each other’s mathematical dialect fluently. To address this situation, the organizers asked the speakers to put a special emphasis on the first, introductory part of their talks. This aspect worked very well.

As a general rule, the organizers asked speakers to talk about specific subjects, not just any nice piece of their research. In some cases, this meant sacrificing hearing about some new results from excellent mathematicians that were further away from the workshop’s main directions.

Mathematics Subject Classification (2010): 20F69, 20E26, 22D40, 37A20, 05C25, 20E05, 22E15, 22E40, 37A50.
Invariant random subgroups, limit multiplicities and Benjamini-Schramm convergence of graphs and Riemannian manifolds. A newly emerging topic is invariant random subgroups (IRS’s). These are conjugacy invariant probability measures on a space of subgroups of a fixed ambient (Lie or discrete) group. It turns out that weak convergence of IRS’s corresponds to Benjamini-Schramm convergence of the quotient spaces. For discrete groups, these are graphs, while for Lie groups, they are locally symmetric spaces.

Tsachik Gelander gave a talk on his joint work with Miklos Abert, Nicolas Bergeron, Ian Biringer, Nikolay Nikolov, Jean Raimbault, and Iddo Samet on Invariant Random Subgroups in higher rank groups. The main result he talked about was to show that for a higher rank simple Lie group $G$, any sequence of locally symmetric $G$-spaces of finite volume converges to the symmetric space of $G$. The proof uses the Stuck-Zimmer theorem. For compact spaces with some natural additional conditions, it also implies the convergence of the normalized Betti numbers. Nicolas Bergeron talked about the same project, but from a representation theoretical point of view. In his talk Limit formulas along BS-converging sequences of $X$-manifolds he showed how Benjamini-Schramm convergence implies the convergence of the Plancherel measures. The methods employed here only work in the cocompact setting. Tobias Finis talked about his joint work with Erez Lapid and Werner Muller where they prove the corresponding theorems on limit multiplicities for nonuniform lattices in $SL_n$.

Lewis Bowen talked about his work on Cheeger constants and $L^2$-Betti numbers. Here he finds a rather unexpected use of Benjamini-Schramm convergence of Riemannian manifolds (using a generalization of the Lück approximation theorem in this setting, due to Elek) to prove a uniform lower bound on the Cheeger constant of certain natural families of discrete subgroups of Lie groups.

Arie Levit gave a talk on a generalization of the intermediate factor theorem to local fields. His work in particular implies that in the presence of property (T), in the above setting, the Stuck-Zimmer theorem holds.

$L^2$ Betti numbers, homology growth and spectral measure.

Wolfgang Lück talked about approximating $L^2$-invariants and homological growth. There is an interesting connection between homology growth, $L^2$ Betti numbers and other invariants, like the cost, the rank gradient and various torsions. Lück gave a thorough survey on the known results and also talked about new directions, like understanding the mod $p$ homology growth and its connections to the other invariants.

Balint Virag gave a talk on his joint work with Lukasz Grabowski on how to defy an old conjecture of Lott and Lück on spectral measure. Note that their result still leaves open the more important general conjecture on determinants to hold.
Russell Lyons gave a talk on L2-Betti numbers, cost, and the free uniform spanning forest. After giving a very good introduction to the subject, he also discussed some new results.

Hanfeng Li gave a talk about when the Fuglede-Kadison determinant is equal to 1. Li (partially in joint works with Chung, Kerr and Thom) made new advances on the topic of entropy for principal algebraic actions of amenable and sofic groups.

Henrik Densing Petersen talked about his joint work on L2-invariants of locally compact groups with Kyed and Vaes and also with Valette. Based on a previous work of Gaboriau, they introduce L2 Betti numbers for locally compact groups in a very general setting, using a von Neumann algebraic approach.

Ergodic theory of group actions. David Kerr gave an introductory talk to sofic entropy with a special emphasis on Bernoulli actions.

François Le Maître gave a talk on his exciting result on the topological rank for full groups. He proved that the topological minimal number of generators for the full group of a pmp equivalence relation always equals the floor of its cost plus one.

Robin Tucker-Drob presented a measure theoretic proof of solid ergodicity for Bernoulli shifts. This is a result of Chifan and Ioana who used von Neumann algebraic tools: now there is an elementary proof. This was one of the talks where the speaker presented the full proof for his result.

Orbit and measure equivalence, rigidity. Uri Bader gave a talk on his joint work with Furman on a new perspective on super-rigidity. They give a new, exciting representation theoretic proof of the Margulis superrigidity theorem, which also leads to natural generalizations. Bader also gave an evening session on the details of the proof. Roman Sauer talked about his joint work with Uri Bader and Alex Furman on L1-measure equivalence of hyperbolic lattices.

Jesse Peterson talked about his work with Thom and another with Creutz on new results on character rigidity. Characters are positive definite, conjugacy invariant functions. They have an intimate connection to von Neumann algebra representations and also to invariant random subgroups (by the work of Vershik). Here rigidity usually means a complete classification of irreducible characters.

Amenable-nonamenable groups. We had three nice talks on amenability-nonamenability.

Kate Juschenko talked about her joint work with Nekrashevych and de la Salle on amenability of groups acting by homeomorphisms on compact spaces. Here they give a very general condition that implies that certain groups are amenable.

Rostislav Grigorchuk talked about his joint work with Benli and Vorobets on random groups of intermediate growth. Here they build an interesting random model that produces natural (uncountable) families of groups of intermediate growth. The speaker analyzed the properties of these random groups.

Mikhail Ershov gave a talk on Tarski numbers. These are the minimal number of pieces needed for a paradoxical decomposition of a nonamenable group. Among
other results, the speaker presented how to use L2 Betti numbers to show that Tarski numbers can get arbitrarily large.

There were also some exciting talks that would be hard to group together by subject.

Denis Osin gave a talk on geometric and analytic negative curvature. The story here is that various people studied various properties of group actions on metric spaces and proved numerous theorems in deep papers. Osin showed that these properties are actually all equivalent to what he calls acylindrically hyperbolic groups.

John Wilson talked about ultraproducts of finite simple groups. He proves an array of results, partially jointly with Thom.

Nir Avni gave a talk on his joint work with Aizenbud on the representation growth of arithmetic lattices. This counts the number of rank $n$ irreducible characters of a given discrete group. The authors find some beautiful and unexpected connections between the representation growth of an arithmetic lattice and the singularities of the moduli space of the corresponding local systems on closed surfaces.

Chen Meiri gave a talk on the Group Large Sieve. This is a new sieve method invented by Rivin, Kowalski, Lubotzky and Meiri that can be used to address the asymptotic properties of random elements of discrete groups. Randomness here is achieved by performing a long random walk with respect to some natural generating set.
Workshop: Group Theory, Measure, and Asymptotic Invariants

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