Abstract. The general topic of the 2013 workshop *Heat kernels, stochastic processes and functional inequalities* was the study of linear and non-linear diffusions in geometric environments: finite and infinite-dimensional manifolds, metric spaces, fractals and graphs, including random environments. The workshop brought together leading researchers from analysis, probability and geometry and provided a unique opportunity for interaction of established and young scientists from these areas.

Unifying themes were heat kernel analysis, mass transport problems and related functional inequalities such as Poincaré, Sobolev, logarithmic Sobolev, Bakry-Émery, Otto-Villani and Talagrand inequalities. These concepts were at the heart of Perelman’s proof of Poincaré’s conjecture, as well as of the development of the Otto calculus, and the synthetic Ricci bounds of Lott-Sturm-Villani. The workshop provided participants with an opportunity to discuss how these techniques can be used to approach problems in optimal transport for non-local operators, subelliptic operators in finite and infinite dimensions, analysis on singular spaces, as well as random walks in random media.

Mathematics Subject Classification (2010): 58J65, 58J35, 60J45, 60K37, 60F17, 53C23.

Introduction by the Organisers

The workshop *Heat kernels, stochastic processes and functional inequalities*, organised by Masha Gordina (University of Connecticut), Takashi Kumagai (RIMS, Kyoto University), Laurent Saloff-Coste (Cornell University), and Karl-Theodor Sturm (University of Bonn) was well attended with over 50 participants from
Australia, Austria, Canada, France, Germany, Israel, Italy, Japan, Luxembourg, Poland, Portugal, United Kingdom, and USA. The program consisted of 26 talks and 5 short contributions, leaving sufficient time for discussions. The general topic of the workshop was the study of linear and non-linear diffusions in geometric environments: finite and infinite-dimensional manifolds, metric spaces, fractals and graphs, including random environments. The workshop was successful in bringing together leading experts in three different major fields of mathematics: analysis, stochastics and geometry. It also provided a unique opportunity for interaction of established and young scientists from these areas. One after-dinner session was devoted to short communications by junior participants of the workshop.

One of the topical focuses of the workshop was related to curvature-dimension bounds, optimal transport, and heat flow in metric measure spaces (Matthias Erbar, Nicola Gigli, Arnaud Guillin, Andrea Mondino) as well as in discrete spaces (Jan Maas). Recent developments in understanding degenerate and singular spaces were presented in talks on generalized curvature-dimension conditions in sub-Riemannian geometry both in finite and infinite dimensions. Classical curvature conditions are problematic in these setting, but Poincaré and other functional inequalities are very useful. Several techniques were discussed including the generalized curvature-dimension condition, optimal transport, stability under concentration limits (Fabrice Baudoin, Tai Melcher, Takashi Shioya).

Methods of Dirichlet forms and heat kernel estimates play key roles in a number of topics discussed during the workshop. In particular, they provide a useful tool in proving various functional inequalities in absence of a well-defined geometry. Analysis on fractals, non-local operators, RWRE etc are some examples of applications. The workshop had talks in a numbers of these topics: non-local operators (Zhen-Qing Chen, Moritz Kassmann), analysis on fractals (Jun Kigami, Naotaka Kajino, Michael Hinz, Ben Hambly), functional inequalities on various metric measure spaces (Martin Barlow, Richard Bass, Thierry Coulhon, Wolfgang Woess). Currently, major research activity is devoted to invariance principles for random conductance models. This also was a topical focus of the workshop (Chris Burdzy, David Croydon, Sebastian Andres). One talk was on the scaling limit of extreme processes for the two dimensional discrete Gaussian Free Field (Marek Biskup). It is linked to potential theory of the Gaussian Free Field via estimations of Green functions.

Two talks were centered on stochastic differential geometry and heat flow in the case when the underlying manifold changes along a geometric flow such as Ricci flow (Ionel Popescu, Anton Thalmaier). Another example of a stochastic version of a deterministic construction was a talk about a stochastic Euler-Poincaré variational principle for a classical Lagrangian on a general Lie group (Ana Bela Cruzeiro). Its classical analogue is Arnold’s picture of the Euler flow as a geodesic on the Lie group of diffeomorphisms.

This diversity of topics and mix of participants stimulated many extensive and fruitful discussions. It also helped initiate new collaborations, in particular for the
young researchers, and strengthen existing ties between researchers in different fields of mathematics.
Workshop: Heat Kernels, Stochastic Processes and Functional Inequalities

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