Abstract. The purpose of this workshop was to bring together researchers with a common interest in the objects mentioned in the title from, respectively, the points of view of toric and tropical geometry, arrangement theory, and geometric group theory.


Introduction by the Organisers

Overview. This Mini-Workshop was organized by G. Denham (London, Canada) and A. Suciu (Boston, USA). The participants were drawn together by overlapping interests in combinatorial constructions in algebraic geometry, topology, and group theory. Several participants were recent Ph.D.s, some of them on their first visit to MFO. In all, there were 17 people attending the mini-workshop (including the organizers), coming from the United States, Canada, Germany, Great Britain, Italy, and Russia.

The meeting allowed us to compare some closely related constructions and find some common ground within the scope of rather varied disciplinary perspectives. The relatively spontaneous format of the meeting made it possible to mix some informal and semi-expository talks and small group discussions with more formal announcements of recent developments, indicated in the abstracts that follow.
Research themes. Some of the mathematical objects of interest included moment-angle complexes and their generalizations, as well as real and complex toric varieties; complex hyperplane and subspace arrangements; fundamental groups such as right-angled Artin groups and arrangement groups; tropical and wonderful compactifications.

One of the themes explored at the meeting was a wider accessibility of ideas from tropical geometry, as applied to the constructions above. The abstract by María Angélica Cueto makes this more precise and provides a quick introduction to the subject. In particular, the construction of De Concini and Procesi’s wonderful compactification of the complement of a union of hyperplanes via toric geometry (an example of Tevelev’s tropical compactifications) was reviewed: the abstract of Eva-Maria Feichtner mentions a solution to a problem posed by Corrado De Concini which was obtained at the workshop. Through informal discussions, the relationship between the Chow rings of the wonderful models with those of the compactifying toric varieties was brought into sharper focus. Furthermore, Diane Maclagan described how methods of tropical geometry could apply to describe the effective cone of the wonderful compactification.

The theory of toric varieties and torus-equivariant topology were implicit ingredients in much of what took place at the meeting. They were, in fact, the focus of the talks on equivariant (co)homology given by Matthias Franz and Hal Schenck. The wonderful models are also closely related to the toric varieties defined by classical root systems, such as the Hessenberg varieties of symmetric, isospectral tridiagonal matrices. These spaces and their cohomology rings admit Coxeter group actions; one approach to understanding their homology uses the representation theory of the reflection groups, together with a subtle comparison with the relevant wonderful compactifications. One of the abstracts by Alex Suciu describes another, totally different approach, based on the topological interpretation of smooth toric varieties pioneered by Davis and Januszkiewicz, and on some recent developments in toric topology.

One of the most fruitful ideas to arise from the theory of hyperplanes arrangements is that of turning the cohomology ring of a space into a family of cochain complexes, parametrized by the cohomology group in degree one, and extracting certain “resonance” varieties from these data, as the loci where the cohomology of those cochain complexes jumps. The abstract of Dan Cohen mentions a solution to a 12-year old conjecture, expressing the ranks of the Chen groups of an arrangement in terms of the dimensions of the components of the resonance varieties. In a more combinatorial vein, the abstract by Mike Falk describes various connections between resonance varieties of arrangements, multinets, Bergman fans, and tropical varieties. Finally, another abstract by Alex Suciu describes a stratification of the Grassmannian of $m$-planes in the second exterior power of a vector space, that keeps track of the corresponding resonance schemes.

Some of the themes from this mini-workshop turned out to overlap with those of the mini-workshop Topology of Real Singularities and Motivic Aspects, which led to some interesting discussions between members of the respective groups. In
particular, Ian Leary had reported on the $\ell^2$-cohomology of hyperplane comple-
ments, and Laurentiu Maxim was able to join this group for an afternoon and
present some complementary results for affine hypersurface complements.

**Concluding remarks.** Spending a concentrated and highly intense week in a
relatively small group allowed for in-depth and continuing conversations, in par-
ticular with new acquaintances. These opportunities (difficult to find at larger
meetings) were enhanced by the diversity of backgrounds of the participants. This
speaks to the fact that the usual, more rigid conference climate was superseded
by an open and creative workshop atmosphere.

There was general agreement that the mini-workshop created an effective and
stimulating research atmosphere. During the week of the workshop, and soon
thereafter, some progress was made in solving old and new problems. The work
initiated at Oberwolfach is continuing now in several research groups. The intense
interactions at the meeting gave rise to new projects, which should start bearing
fruit in the not too distant future.
Mini-Workshop: Cohomology Rings and Fundamental Groups of Hyperplane Arrangements, Wonderful Compactifications, and Real Toric Varieties

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