Rough Paths and PDEs

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Abstract. The purpose of the Oberwolfach workshop "Rough Paths and PDEs" was to bring together these researchers, both young and senior, with the aim to promote progress in rough path theory, the connections with partial differential equations and its applications to numerical methods.


Introduction by the Organisers

The rough path theory, initiated by T. Lyons (workshop participant) in the nineties has had a profound influence on stochastic analysis; its single most important results is that solutions to stochastic differential equations can be solved pathwise and that the solution map is continuous (even locally Lipschitz) in rough path metric. This continuity property has since become the key in many striking applications, ranging from the Stroock-Varadhan support theorem in its as-of-yet strongest form to a new understanding of Hörmander's theory without Markovian structure. Much of this has been summarized in a recent monograph of Friz (workshop organizer). By applying and extending rough paths ideas to (stochastic) partial differential equations, a fruitful connection was established between the stability of (stochastic) flows in rough path sense and the stability properties of viscosity solutions to PDEs. In particular, large classes of SPDEs are reduced to (deterministic) partial differential equations driven by rough signals. This is closely related to the (essentially pathwise) Lions-Souganidis theory of stochastic viscosity solution. Souganidis was a participant at the workshop. A related set of new ideas is to introduce rough path stability in the context of backward (doubly)
stochastic differential equations (BSDEs): in a sense this amounts to non-linear Feynman-Kac formulae for rough partial differential equations. BSDEs have been introduced in the eighties by another workshop participant, Shige Peng. Another important application: stochastic filtering is concerned with the estimation of the conditional law of a Markov process, given observations of some function of it. Using the tools provided by rough paths one can show that it is essential to measure not just the observation process but also its associated area process. In other words, filtering has now become an outlet for rough paths developments. The following workshop participants are active in this area Diehl, Oberhauser, Friz and Crisan. Lastly, rough paths theory has had an importance influence in the area of numerical approximations of solutions of PDEs deterministic as well as stochastic. Litterer, Lyons and Crisan work on this topic.

The Mathematisches Forschungsinstitut Oberwolfach offered the ideal environment to enhance the synergy between the participant experts working in these related areas.
Workshop: Rough Paths and PDEs

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