Mathematisches Forschungsinstitut Oberwolfach

Report No. 34/2012

DOI: 10.4171/OWR/2012/34

Discrete Differential Geometry

Organised by
Alexander I. Bobenko, Berlin
Richard Kenyon, Providence
Peter Schröder, Pasadena
Günter M. Ziegler, Berlin

July 8th– July 14th, 2012

Abstract. This is the collection of extended abstracts for the 24 lectures and the open problems session at the third Oberwolfach workshop on Discrete Differential Geometry.

Mathematics Subject Classification (2010): 52-xx, 53-xx, 57-xx.

Introduction by the Organisers

Discrete Differential Geometry is a very productive research area where graph theory, analysis, integrability, and geometry interact and contribute to the construction and understanding of discrete models for differential geometric situations and structures. It also plays a very important role in applications, to graphics and simulations of PDE.

This was the third Discrete Differential Geometry conference at Oberwohlfach. The subject has evolved significantly since its beginning a decade ago. This year's conference highlighted advances in new areas: in discrete exterior calculus and cluster algebras in geometry, as well as in some older ones: discrete uniformization, polyhedra, applications to PDE.

The workshop featured many talks around the subject of discrete exterior calculus. The main idea of discrete exterior calculus is to find the right adaptation of the classical notions of forms, exterior differentiation, Hodge decomposition, etc. to functions on cell complexes, with the goal being to do classical analysis using discrete approximations to continuous objects. The talks by Chelkak, von Deylen, Günther, Hildebrandt, Skopenkov, Stern all fit in this category. From the variety

of techniques presented we can only say that the subject is still under discussion, and that a global framework is still to be found.

Another new and exciting direction is the integrability and cluster structure of discrete geometric mappings. Here we heard talks by Doliwa, Goncharov, Suris, Tabachnikov on connections between various discrete systems and cluster algebras/varieties or other integrable structures. This area seems ripe for further exploration, in particular since we don't understand what features these models have in common, and what consequences the cluster structure may have. In particular the cluster structure allows one to introduce quantization which may yield important new avenues of research.

There were a few talks about polyhedra (by Adiprasito, Izmestiev, Schlenker) and versions of discrete uniformization (by Sechelmann, Stephenson, Sullivan). Although these are more well-studied areas the new ideas presented open new opportunities for further research.

Finally there were a few talks about discretizations of PDE: by Crane, Lessig, Schief, Schumacher, Hoffmann, Vouga. Here we include applications to graphics: mapping textures to surfaces and smoothing using conformal maps is one common theme. This area of PDE applications continues to be an important source of inspiration for theoretical advances in discrete differential geometry: our goal is to be able to model PDEs, and often finding the right discretization makes a huge difference in efficiency. Furthermore some systems have discretizations which are in some sense more natural than their continuous counterparts (in the sense that there is more mathematical structure).

The organizers are grateful to all participants for all the lectures, discussions, and conversations that combined into this very lively and successful workshop – and to everyone at Research Institute in Oberwolfach for the perfect setting.

Workshop: Discrete Differential Geometry

Table of Contents