Abstract. Applied harmonic analysis and sparse approximation are highly active research areas with a lot of recent exciting developments. Their methods have become crucial for a wide range of applications in technology and science, such as signal and image processing. Understanding of the underlying mathematics has grown vastly. Interestingly, there are a lot of connections to other fields, such as convex optimization, probability theory and Banach space geometry. Yet, many problems in these areas remain unsolved or even unattacked. The workshop intended to bring together world leading experts in these areas, to report on recent developments, and to foster new developments and collaborations.

Mathematics Subject Classification (2000): 42-XX, 65Txx, 94Axx, 65K05, 15A52.

Introduction by the Organisers

The workshop *Applied Harmonic Analysis and Sparse Approximation* was organized by Ingrid Daubechies (Durham), Gitta Kutyniok (Berlin), Holger Rauhut (Bonn) and Thomas Strohmer (Davis). This meeting was attended by 53 participants from about 10 countries and 3 continents.

Applied Harmonic Analysis has seen enormous developments in the last three decades. Its tools and methods have turned out to be crucial in a wide range of signal and image processing applications, in numerical algorithms for the solution of operator equations, and in inverse problems. In addition, mathematical understanding of the underlying harmonic analysis has grown vastly – with research sometimes driven by the needs of various applications and its results, in turn leading to renewed interplay with applications. Sparse approximation and
compressed sensing constitute a recent development with clear roots in applied harmonic analysis as well as in other areas such as statistics and optimization. It has been the focus of an intense research activity in recent years, and – despite its young age – has become already a fairly mature mathematical discipline. This does by far not mean that all problems are solved – only that the field is past its childhood: There are still various important and sometimes deep open problems in applied harmonic analysis and sparse approximation. Compressed sensing has moreover shown intriguing connections to the fields of random matrix theory and convex optimization.

One main focus of research in applied harmonic analysis is on the development of novel structured dictionaries which exhibit specific properties – for example, optimal sparse approximation – for desired classes of mathematical objects. This has led to the recent introduction of curvelets, shearlets, and other representation systems, which have already impacted both theoretically oriented questions such as sparse expansions of Fourier integral operators and application orientation areas such as image processing. There remains, however, a wide range of fundamental open questions; for instance, the understanding and/or characterization of associated function spaces, which is essential for a mathematically satisfactory analysis of such representations systems.

One key property that ensures the occurrence of sparse expansions is the redundancy of these systems. This is the focus of the research area of frame theory, which studies various aspects of redundancy as a mathematical concept. Frame theory already impacted the whole area of applied harmonic analysis and sparse approximation significantly; yet, surprisingly, a fundamental understanding of redundancy measures is just at its beginning.

Sparsity has become a very important concept in recent years in applied mathematics, especially in mathematical signal and image processing. The key idea is that many types of functions and signals arising naturally in these contexts can be described using only a small number of significant terms in a suitable basis or frame, often a wavelet basis, a Gabor frame or a shearlet frame. This is essentially the reason why many lossy compression techniques such as JPEG or MP3 work so well. Quite interestingly, sparsity is useful not only for compression purposes. The new field of compressed sensing predicts that sparse high-dimensional signals can be recovered efficiently from what would previously have been considered highly incomplete measurements. This discovery has led to a fundamentally new approach to certain signal and image recovery problems, which can in fact be regarded as a paradigm change. Remarkably, the main constructions for good measurement matrices known so far are random, and the mathematical research in compressive sensing uses also tools, sometimes quite sophisticated, from probability theory and the geometry of Banach spaces.

The workshop featured 31 talks, thereof 9 longer overview talks. Moreover, a session of very short presentation of 3 minutes took place on Monday, which we called the 3 Minutes of Fame (following Andy Warhol’s concept of 15 minutes of fame). Every participant had the possibility to contribute to this session, and this
experiment worked out very well. In particular, it provided a quick overview on what the participants are presently working or would like to discuss with other participants.

Some highlights of the presentations include:

- **Super-resolution:** Emmanuel Candés reported on a new approach to super-resolution based on convex optimization. Quite surprisingly and in contrast to compressive sensing, this theory does not require randomness and demonstrates that one can extrapolate the high-frequency content of a signal from only low-frequency information.

- **Co-sparsity:** A new direction in sparse approximation uses the notion of co-sparsity. Rather than requiring that has a sparse representation with respect to a basis or frame, one requires that $\Omega x$ is sparse, where $\Omega$ is a so-called analysis operator. This approach has advantages in certain signal processing tasks. An overview talk on this topic was given by Michael Elad, and this idea appeared also in some of the shorter talks, e.g. by Emily King and Ignace Loris.

- **Signal separation:** Very accurate bounds for the problem of separating two signals “of different nature” via convex programming were presented in the talk of Joel Tropp. This rather general theory covers several situations including that both vectors are sparse in different bases and that one is a sparse matrix and the other one a low rank matrix.

- **Phase retrieval:** Thomas Strohmer presented a new method to recover a signal $x$ based on knowledge of only the absolute values of some coefficients $|\langle a_k, x \rangle|$ with respect to some vectors $a_k$. This problem arises for instance in diffraction imaging, where one measures the absolute value of the Fourier transform. The method builds on ideas from low rank matrix recovery and uses a convex optimization program to find $x$. First results were reported, but many open problems remain. A shorter talk on phase retrieval was presented by Radu Balan.

Further new interesting developments include the use of sparsity in flocking (Massimo Fornasier), new constructions of shearlets, results for corresponding function spaces and applications in image segmentation (Stephan Dahlke, Gabriele Steidl, Gerd Teschke; Philipp Grohs), new results around the restricted isometry property (Bernhard Bodmann; Felix Krahmer) as well as new algorithms for sparse recovery (Ignace Loris; Sergey Voronin).

The organizers would like to take the opportunity to thank MFO for providing support and a very inspiring environment for the workshop. The magic of the place (as coined by one of the participants) and the pleasant atmosphere contributed greatly to the success of the workshop.
Workshop: Applied Harmonic Analysis and Sparse Approximation

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