Invariants in Low-Dimensional Topology and Knot Theory

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Abstract. This meeting concentrated on topological invariants in low dimensional topology and knot theory. We include both three and four dimensional manifolds in our phrase “low dimensional topology”. The intent of the conference was to understand the reach of knot theoretic invariants into four dimensions, including results in Khovanov homology, variants of Floer homology and quandle cohomology and to understand relationships among categorification, topological quantum field theories and four dimensional manifold invariants as in particular Seiberg-Witten invariants.

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Introduction by the Organisers

The purpose of this conference was to bring together people working in low-dimensional topology, both in knot theory and 3-manifold theory and in 4-manifold theory.

Here is a short comment on the combinatorial topology side of the topics. In 1969 John H. Conway published a version of the Alexander polynomial that involves nothing more than a recursion on diagrams controlled by a “skein formula” that expresses the difference between the polynomial for a knot with a given crossing, the same diagram with a switched crossing and the same diagram with the crossing replaced by connecting arcs that do not cross (a smoothing of the crossing). This remarkable reformulation of the Alexander polynomial remained a mystery for some years. In the late 1970’s people became interested in this relation again and, among others, Kauffman wrote a paper explaining the skein
relation approach of Conway in terms of the Seifert pairing of the knot. In the early 1980’s Kauffman found another model of the Alexander-Conway Polynomial as state summation related to Alexander’s original definition using a determinant of a matrix associated with the link diagram. Then in 1983, Vaughan Jones found a new and powerful polynomial invariant of knots and links that was quite different from the Alexander polynomial, but also satisfied a skein relation. This discovery of the Jones polynomial quickly led to a number of other skein-type invariants – the Homflypt polynomial and a two-variable Kauffman polynomial. Also Kauffman found a state sum model for the original Jones polynomial. After this initial combinatorial revolution in the knot theory, there came a big influx of algebra, first via von Neuman algebras and the Temperley Lieb algebra from Jones himself, then Hecke algebras and quantum groups (deformations of classical Lie algebras) and Hopf algebras with the work of Reshetikhin and Turaev. Then quantum field theory entered the picture with the work of Edward Witten and this led to the development of new invariants of three-manifolds, the formulation of Vassiliev invariants, work of Birman, Lin and Bar Natan and a mix of research problems that continues to the present day. In the 1990’s Kauffman and Goussarov, Polyak and Viro introduced virtual knot theory a generalization of classical knot theory to knots and links in thickend surfaces that has a simple diagrammatic extension from classical knot diagrams. Virtual knot theory continues in a very active way to the present day with contributions from many people and a first book on the subject by Manturov and Ilyutko, containing significant recent advances by Manturov and collaborators. In 1999 Misha Khovanov discovered an extension of the Kauffman bracket state sum model for the Jones polynomial to a graded homology theory such that the coefficients of the Jones polynomial become Euler characteristics of graded parts of the homology. The Khovanov homology of a knot is more powerful than the Jones polynomial of that knot and in fact it was shown in 2008 that the Khovanov homology detects the unknot, a property that is still unknown for the Jones polynomial. This “categorification” of the Jones polynomial was followed by a quite different categorification of the Alexander-Conway polynomial in the work of Oszváth and Szabó, and this work led to astonishing results such as a homological method to find the Seifert genus of a knot and, in both cases of these theories, a bridge between three manifolds and four manifolds. This sketch indicates the background of our conference on the side of combinatorial topology.

There were 51 participants, and 42 speakers among them. Participants without talks presented their results in various private communications during the discussion time or in the evening at the workshops or in an unofficial manner.

Several talks were organized for the whole audience; the other talks were held in two parallel sessions.

Nevertheless, all participants could share their results with everyone in formal or less formal workshops organized every day in the evening time. Research reports of the majority of participants were posted on the wall as well as on the conference webpage.
The main topics of the conference were:

- Recognition of the Unknot
- Virtual Knot Theory and Parity Theory
- Cobordisms and Concordance of Knots
- Finite-type invariants
- Heegaard-Floer Homology
- Exotic structures and Corks in 4-manifold Theory
- Seiberg-Witten Invariants, Gauge Theory
- 2-knots and their diagrams
- Khovanov homology theory
- Braid Theory
- Unknotting numbers and related topics in classical and virtual knot theory
- Quandles and Related Structures in Knot Theory
- Knot Mutations
- Knots and DNA
- Contact topology
- Topological Methods in Combinatorial Group Theory
- Calabi-Yau Manifolds
- Fibred Manifolds

In addition to the talks, three workshops were organized during the conference. A workshop on Virtual Knot Theory and parity in Low-Dimensional Topology was organized by V.O. Manturov. It was devoted to further applications of parity theory as well as to various unsolved problems, and others contributed to the discussion.

One workshop was devoted to the result of Chad Musick on the recognition of the unknot in a polynomial time.

One workshop was organized by Scott Carter on various algebraic generalizations of quandles possessing distributivity and associativity properties leading to invariants of knots, 2-knots and trivalent graphs.
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