Mini-Workshop: Hypergraph Turán Problem

Organised by
Penny Haxell, Waterloo
Dhruv Mubayi, Chicago
Oleg Pikhurko, Coventry
Tibor Szabó, Berlin

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Abstract. This mini-workshop focused on the hypergraph Turán problem. The interest in this difficult and old area was recently re-invigorated by many important developments such as the hypergraph regularity lemmas, flag algebras, and stability. The purpose of this meeting was to bring together experts in this field as well as promising young mathematicians to share expertise and initiate new collaborative projects.

Mathematics Subject Classification (2000): 05D05, 05C35.

Introduction by the Organisers

The mini-workshop Hypergraph Turán Problem, organised by Penny Haxell (Waterloo), Dhruv Mubayi (Chicago), Oleg Pikhurko (Coventry), and Tibor Szabó (Berlin) was held 8–14 April 2012. This meeting was attended by 17 participants from 6 different countries. The purpose of the mini-workshop was to bring together researchers of different backgrounds and seniority so that they can communicate about recent developments, share their expertise, and continue or initiate collaborative projects. In particular, the organizers invited quite a few researchers who are early in their careers; for example, for 4 participants it was the first time that they were at the MFO.

The schedule was designed to give the participants ample free time for collaboration and discussions. There were 15 talks in total (in mornings) and a problem session (on Monday afternoon). The format of talks varied from a general introduction to some important aspect (such as the 2-part lecture by John Talbot on flag algebras) to a short communication of a recent result.
The hypergraph Turán problem is about 70 years old. The basic question here is to estimate the Turán function \( ex(n, F) \) which is the maximum number of edges in a hypergraph \( G \) on \( n \) vertices that does not contain the given forbidden \( k \)-graph \( F \). This fundamental problem, relating global and local parameters, is notoriously difficult and wide open in general. For example, the famous conjecture of Turán that \( ex(n, K^4_3) = \left( \frac{5}{9} + o(1) \right) \binom{n}{3} \), where \( K^4_3 \) denotes the complete 3-graph on 4 vertices, is still open despite the $1000 prize of Erdős. Nonetheless this area is a great success if one judges by the wealth of ideas, methods, and connections that were discovered during the decades of active attempts. The talks presented at the mini-workshop reflected this variety very well, relating Turán-type questions to Ramsey theory, quasi-randomness, extremal problems on hypercubes, decomposition theorems, matchings, \( H \)-factors, and counting independent sets in hypergraphs.

One important general development that was motivated by the hypergraph Turán problem was the semidefinite method of Razborov built upon his flag algebras framework. A number of new results obtained by this method were presented at the mini-workshop; also, some participants investigated whether the semidefinite method may apply to extremal problems for other structures such as permutations or monochromatic arithmetic progressions.

The mini-workshop was quite active in terms of ongoing and new collaboration. The joint research projects that were carried out during the workshop included applying flag algebras to permutation densities (Dan Král’ and Oleg Pikhurko), co-degree Turán densities (Oleg Pikhurko and Emil Vaughan), hypergraph Ramsey problems for loose cycles (Alexandr Kostochka and Dhruv Mubayi), tight cycles (Dhruv Mubayi and Vojtĕch Rödl) and extremal hypergraphs for packing and covering (Penny Haxell and Tibor Szabó).

The mini-workshop was a great success. We are very grateful to the Oberwolfach Mathematical Institute for providing such a stimulating and productive environment.
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Table of Contents

John Talbot (joint with Rahil Baber)
Turán densities and stability via Razborov’s flag algebra method ............ 1153

Tao Jiang (joint with Axel Brandt, Robert Seiver)
Turán numbers of expanded hypergraphs .................................. 1155

Problem Session ............................................................... 1156

Imre Leader (joint with Béla Bollobás, Claudia Malvenuto)
Daisies ................................................................. 1157

Po-Shen Loh (joint with Hao Huang, Benny Sudakov)
The size of a hypergraph and its matching number ......................... 1158

Yury Person (joint with Peter Allen, Julia Böttcher, Hiệp Hàn and
Yoshinari Kohayakawa)
Powers of Hamiltonian cycles in pseudorandom graphs .................. 1160

Alexandr Kostochka (joint with Dhruv Mubayi, Jacques Verstraete)
Hypergraph Ramsey Numbers: Triangles versus Cliques ................. 1162

Oleg Pikhurko
On Possible Turán Densities ............................................... 1164

Julia Wolf
Analogies and differences: colouring elements of \(\mathbb{Z}_p\) and edges of \(K_n\) ... 1166

Peter Keevash (joint with Benny Sudakov, Jacques Verstraëte)
On a conjecture of Erdős and Simonovits ................................. 1168

Daniel Král’ (joint with Oleg Pikhurko)
Quasirandom permutations .................................................. 1168

József Balogh (joint with Robert Morris, Wojciech Samotij)
Independent sets in hypergraphs ......................................... 1170

Zoltán Füredi
Linear trees in uniform hypergraphs ..................................... 1175

Emil R. Vaughan (joint with Victor Falgas-Ravry, Oleg Pikhurko)
Codegree densities of 3-graphs .......................................... 1178

Jan Hladký (joint with János Komlós, Diana Piguet, Miklós Simonovits,
Maya J. Stein, Endre Szemerédi)
Loebl-Komlós-Sós Conjecture and structure of possibly sparse graphs ... 1179