Mini-Workshop: Thermodynamic Formalism, Geometry and Stochastics

Organised by
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Abstract. Thermodynamic formalism and all its branches and applications in conformal dynamics, probability theory, stochastics and fractal geometry represent highly important fields in modern Mathematics, which are currently very active and rapidly growing. The workshop brought leading experts in these fields together with junior researchers to provide them with the opportunity to exchange their knowledge and experience. It led to various new insights as well as promising new research collaborations.


Introduction by the Organisers

The workshop Thermodynamic Formalism, Geometry and Stochastics, organised by Bernd Otto Stratmann (Bremen), Mariusz Urbański (Denton, Texas) and Anna Zdunik (Warsaw), was held January 1st – January 7th, 2012. This meeting was well attended with 17 participants with broad geographic representation from almost all continents. Every participant gave a 45 minutes presentation and additionally, there was a round table discussion about thermodynamic formalism as well as an Open Problem Session to which almost every participant contributed. The workshop was a splendid blend of researchers with various different backgrounds in thermodynamic formalism, conformal dynamics, probability theory, stochastics and fractal geometry. These areas are central in the theory of dynamical systems. They have their origins in the pioneering work of Ruelle, Sinai, Bowen, Dobroushin and others, who from the late sixties until the mid seventies
adapted key methods from statistical physics, in particular the theory of gas lattices, to the context of continuous dynamical systems on compact metric spaces. It was also during this period that important concepts such as topological pressure, the variational principle, equilibrium states and Gibbs states made their decisive breakthrough.

The first areas in which the principles of thermodynamic formalism have successfully been applied include Axiom A diffeomorphisms and smooth expanding endomorphisms of Riemannian manifolds and the foundations for these important applications were laid by Bowen and Ruelle. Since then, thermodynamic formalism has flourished in many different directions and found fruitful applications in various fields in Pure and Applied Mathematics. The recent book *Conformal Fractals: Ergodic Theory Methods* by Przytycki and Urbański, which both were present at the workshop, provides a systematic account of the current state of the art.

In conformal dynamics and fractal geometry, the relevance of thermodynamic formalism became apparent through groundbreaking work of Rufus Bowen, which established a relationship between the Hausdorff dimension of the limit set of a quasi-Fuchsian group and the unique zero of the associated pressure function. This was expressed in one formula, now called Bowen’s formula. Shortly after the appearance of this formula, it became clear that Bowen’s approach is also applicable in many other situations, most notably in the study of Julia sets of conformal expanding maps. Also, thermodynamic formalism has been employed in differential geometry to derive deep new insights into the nature of geodesic flows on compact Riemannian manifolds with negative curvature. In number theory, prominent applications of thermodynamic formalism were given in the realm of continued fractions and the Gauss map. This has been done, for instance, in a series of papers by D. Mayer and also via the theory of conformal graph directed Markov systems with an infinite set of edges, whose general theory has been developed by Mauldin and Urbański. A further generalisation of the latter theory to the case in which additionally the set of vertices is infinite has been obtained in joint work by Stratmann and Urbański. Moreover, a closely related multifractal analysis of the Gauss map and the Farey map has been obtained in a series of papers by Kesseböhmer and Stratmann. The graph directed approach to continued fractions has also led to various important theorems about real numbers whose continued fraction expansions have entries restricted to some fixed infinite subset of positive integers.

Besides the above-mentioned applications of thermodynamic formalism in the study of limit sets of Kleinian groups, thermodynamic formalism has also had great impact in the rigorous exploration of fractal phenomena of Julia sets of rational functions on the Riemann sphere. For the most classical of these maps, namely those which are expanding on their Julia sets, the results for conformal expanding repellers can be applied directly. Moreover, the case of parabolic rational maps has been almost completely dealt with in a cycle of classical papers by Denker and Urbański. The non-recurrent case has been extensively discussed by Urbański, whereas the important class of Collet-Eckmann maps has been explored.
thoroughly by Graczyk and Smirnov, as well as by Rivera-Letelier and Przytycki. The work of Denker and Urbanski has also been significant in the comprehensive further development of the theory of equilibrium states for Hölder continuous potential functions, which also served as one of the main motivations for the important work of Haydn, as well as for the very recent research of Urbanski and Zdunik concerning fine inducing.

There are numerous further areas where thermodynamic formalism has turned out to be an indispensable tool. For instance, thermodynamic formalism has always been fruitfully inspired by multifractal formalism, which is part of fractal geometry and a special case of thermodynamic formalism. Moreover, in finer studies of parabolic phenomena, recent work by Thaler and Zweimüller, using thermodynamic formalism, has shed new and surprising light on Darling-Kac type theorems. Similarly, recent work of Melbourne significantly clarifies statistical properties of these parabolic maps. Another large area where thermodynamic formalism has been applied to, is the field of random dynamical systems. Here, the work of Rugh has to be mentioned, which is based on pioneering research by Kifer, Bogenschütz and Gundlach. However, this area is far from being complete. For instance, Simmons works on random iterates of general rational functions and Hölder continuous potentials, and V. Mayer, Skorulski and Urbanski are currently working on random parabolic Cantor sets as well as random transcendental meromorphic functions. Also, thermodynamic formalism for transcendental entire and meromorphic functions has flourished since the seminal work of Barański, which in turn has been intensively developed further by Kotus, V. Mayer, Urbanski and Zdunik.
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