Abstract. This workshop concentrated on partial differential equations involving stationary and evolving surfaces in which geometric quantities play a major role. Mutual interest in this emerging field stimulated the interaction between analysis, numerical solution, and applications.

Mathematics Subject Classification (2000): MSC: 35-XX, 49-XX, 65-XX.

Introduction by the Organisers

The workshop Geometric partial differential equation: Theory, Numerics, and Applications, organized by Gerd Dziuk (Freiburg), Charles M. Elliott (Warwick), Gerhard Huisken (Golm), and Ralf Kornhuber (Berlin) was held November 28–December 2, 2011. The scientific program consisted of 23 plenary talks with a good mix of slide and blackboard presentations and a 'Young researcher’s session' on Wednesday evening. About 50 participants from America, Asia, and Europe represented the leading experts both from analysis and numerical analysis of partial differential equations (pdes) involving stationary and evolving surfaces and interfaces. The diversity of participants provided a wealth of new perspectives on this emerging field. Mutual interaction of analysis and numerics led to lively discussions after the talks, created a stimulating atmosphere throughout the conference, and gave rise to new, unexpected cooperations.

The focus was on partial differential equations of, on, and into Stationary and evolving surfaces and interfaces. Since the last decades of the preceding century,
mean curvature flow played a major role in geometric pdes. As one of the highlights, Klaus Ecker reviewed fundamental results on the Cauchy problem in the graph case which, in contrast to the standard heat equation allows for a smooth solution for all times without any growth conditions. Matteo Novaga discussed the long-time behavior of mean curvature flow in heterogeneous media. Using variational arguments rather than maximum principles, he showed convergence to traveling wave solutions. Harald Garcke reported on existence results for clusters of hypersurfaces evolving by mean curvature that meet under suitable angle conditions. Many grains with boundaries moving by mean curvature were considered by Selim Esedoglu. His highly optimized level set algorithm allows for large scale simulations of recrystallization for physically relevant parameter values. The motion of the interface of two immiscible fluids depends both on surface tension effects and on the dynamics in the bulk. Existence of strong solutions for an incompressible Navier-Stokes/Mullins-Sekerka system were presented by Helmut Abels. Anisotropic versions of mean curvature flow was considered by Carsten Gräser who concentrated on the stability analysis of various time discretizations of anisotropic phase field equations and the efficient and robust solutions of the resulting spatial problems by recent non-smooth Newton multigrid methods. Crystalline mean curvature flow is generated by anisotropic interfacial energies which are singular in the sense that the associated Frank diagram is no longer strictly convex. In his talk Yoshihazu Giga discussed polygonal motion, together with a variational and a viscosity approach to appropriate solution concepts and corresponding numerical techniques. Maurizio Paolini introduced the bidomain model, a system of two reaction-diffusion equations, which turned out to be formally asymptotic to non-convex, anisotropic mean curvature flow, and presented some numerical experiments.

The notion of bending energy similar to classical Kirchhoff plate theory gives rise to Willmore or Hellfrich energies which play a significant role in the macroscopic modeling of biomembranes. Klaus Deckelnick considered a conforming semi-discretization in space of the Willmore flow, i.e. the associated $L^2$ gradient flow of the Willmore energy in the graph case, and proved optimal error estimates for the resulting method of lines. Björn Stinner provided a numerical study of $H^1$-gradient flow for the Willmore functional using a mixed formulation consisting of quadratic and linear surface elements. A network of curves driven by elastic flow, a variant of Willmore flow, was considered by Robert Nürnberg. A striking feature of his algorithm is to use tangential motion to preserve well-distribution of mesh points throughout the evolution.

Ricci flow plays a crucial role not only in the celebrated proof of Poincaré’s conjecture but also in general relativity. James Isenberg, one of the pioneers of this field, gave a survey on numerics, matched asymptotics, and analysis of degenerate neckpinches in Ricci flow, while Hans Fritz presented an innovative definition of discrete Ricci curvature on triangulated hypersurfaces of arbitrary dimension based on a suitable weak formulation.
Partial differential equations on surfaces. In spite of considerable progress during the last years, the numerical analysis of pdes on surfaces is still its infancy. The construction of parametrized cubical grids from an arbitrary tetrahedral mesh was presented by Konrad Polthier. Andrea Bonito reported on the state of the art of adaptive finite element methods for Laplace-Beltrami problems. Christian Lubich investigated implicit Runge-Kutta methods as applied to the ODEs arising from the spatially discrete Evolving Surface Finite Element Method, showing that the order of convergence is inherited from the classical case. Dietmar Kröner presented the ideas of a proof for existence and uniqueness of entropy solutions of nonlinear conservation laws on moving surfaces. Applications of adaptive finite elements to the numerical solution of the Einstein equations were given by Michael Holst. Oliver Rinne presented an approach to Einstein equations on constant mean curvature surfaces based on compactified coordinates. Many engineering problems involving, e.g., the motion of foams, grain growth or the evolution of multicellular structures give rise to coupled models for pdes on moving surfaces. James Sethian discussed new approaches to this class of problems and showed various applications.

Partial differential equations into surfaces. Partial differential equations with non-convex constraints often can be regarded as pdes into surfaces. For example, the elastic bending of a thin plate leads to a minimization problem with isometry constraints. Sören Bartels introduced and analyzed finite element approximations of this problem and reported on recent numerical computations. Oliver Sander suggested an intrinsic approach to pdes into surfaces based on piecewise polynomial approximation along geodesics. An application of geodesic interpolation to topology preserving shape morphing and related problems in geometry processing were presented by Martin Rumpf.

The Young researcher’s session on Wednesday evening took place in a relaxed atmosphere with a glass of wine provided by the organizers. Two PhD students and two post-docs took the opportunity to present their recent work to an international audience and discuss possible future developments and perspectives.
# Workshop: Geometric Partial Differential Equations: Theory, Numerics and Applications

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