Abstract. The aim of this workshop was to bring together specialists in the area of stochastic dynamical systems and stochastic numerical analysis to exchange their ideas about the state of the art of approximations of stochastic dynamics. Here approximations are considered in the analytical sense in terms of deriving reduced dynamical systems, which are less complex, as well as in the numerical sense via appropriate simulation methods. The main theme is concerned with the efficient treatment of stochastic dynamical systems via both approaches assuming that ideas and methods from one ansatz may prove beneficial for the other. A particular goal was to systematically identify open problems and challenges in this area.


Introduction by the Organisers

The workshop Dynamics of Stochastic Systems and their Approximation, organised by Evelyn Buckwar (Edinburgh), Barbara Gentz (Bielefeld) and Erika Hausenblas (Leoben) was held August 21st – August 27th, 2011. This meeting was well attended with 17 participants from 6 countries, with one participant having been awarded a US Junior Oberwolfach Fellowship to attend the workshop.

Stochastic modelling has become a standard tool in many areas of science and engineering, in fields as diverse as finance, molecular dynamics, neuroscience, laser physics, hydrogeology or climate research. Often, the mathematical description of real world phenomena is in terms of stochastic ordinary differential equations, stochastic delay differential equations or stochastic partial differential equations.
Although the aims of stochastic modelling are as manifold as the classes of equations and application areas, in several application areas the interest in stochastic modelling is in particular focussed on understanding the dynamical behaviour of the stochastic systems. Much of the research in, e.g., neuroscience or climate modelling is devoted to noise-induced or noise-related phenomena in the dynamical behaviour of the system.

Several approaches and techniques to describe and analyse the dynamics of stochastic systems exist. The mini-workshop aimed at bringing together people from different areas of expertise and the given talks have been grouped into the following themes:

- Stochastic dynamics in the Sciences,
- Numerics for stochastic dynamical systems,
- Reduced dynamics,
- Stochastic partial differential equations.

Participants had been asked to prepare a review talk on a specific subtopic fitting one of these themes, thus providing a basis for the subsequent productive discussions. In the first theme the following participants gave talks: Nils Berglund, Evelyn Buckwar, Barbara Gentz, Peter Imkeller, Rachel Kuske, Tony Lelièvre and Kevin Lin. The second theme was covered by Erika Hausenblas, Peter Kloeden, Andreas Neuenkirch and Tony Shardlow. Martin Riedler and Richard Sowers spoke about the third theme, and the final theme was addressed by Dirk Blömker, Anne de Bouard, Zdzislaw Brzezniak and Sandra Cerrai.

In the course of the mini-workshop, a number of challenging open questions were identified, examples are given by:

- Different concepts of numerical treatment of long-time dynamics of stochastic systems,
- Asymptotic dynamics of delay equations and reaction-diffusion equations with noise,
- Rigorous limit procedures between descriptions of a multi-scale system by continuous or discrete state spaces, depending on the relevant scale. This kind of problem arises, e.g., in molecular dynamics and neuroscience.

The organisers thank the Mathematisches Forschungsinstitut Oberwolfach for providing an inspiring setting for this mini-workshop.
Mini-Workshop: Dynamics of Stochastic Systems and their Approximation

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