Abstract. The major topics discussed in this workshop were the Feichtinger conjecture and related questions of harmonic analysis, the corona problem for the ball $B^n$, the weighted approximation problem, and questions related to the model spaces, to multipliers, (hyper-)cyclicity, differentiability, Bezout and Fermat equations, traces and Toeplitz operators in different function spaces. A list of open problems raised at this workshop is also included.


Introduction by the Organisers

The “Paving Conjecture” of Kadison–Singer claims that given any linear contraction whose diagonal matrix elements are zero with respect to a given basis, there is a partition of the basis in a universal number of pieces such that the compression of the operator to the span of each piece of the partition (or the minor matrix generated by the piece, if one prefers) has norm bounded above by one half. Recently, it became clear that this conjecture is equivalent to several others including a conjecture generalizing the Bourgain–Tzafriri Restricted Invertibility Theorem and the Feichtinger conjecture, which claims that every frame in a Hilbert space is a finite union of Riesz basis sequences. In many reproducing kernels Hilbert spaces like the Hardy space and its model subspaces, the de Branges spaces, the Smirnov spaces, the Bergman space, the Fock space, and their weighted analogs, questions on frames and Riesz basis sequences of reproducing kernels are reformulated as deep problems concerning uniqueness, interpolation and sampling properties of
analytic functions. See, for example, the work by Seip–Wallstén on the Fock space and the 2007 paper by Borichev–Dhuez–Kellay on the weighted Fock spaces and the references therein. Frequently, such problems are related to harmonic analysis applications, from wavelet theory, time-frequency analysis (Gabor frames), operator theory (spectral theory of Toeplitz operators), and systems theory (via, for example, spectral theory of Hankel operators).

Therefore, the Feichtinger conjecture was one of the main topics of the workshop. In the first talk of the workshop, P. Casazza described numerous equivalent reformulations of the Kadison–Singer problem and proposed a method to construct a counter-example using the Laurent operators. V. Vasyunin described his results on trace $H^\infty$-algebras giving a negative answer to a stronger form of the Bourgain–Tzafriri restricted invertibility conjecture. N. Lev presented his results on the Riesz bases of exponentials with restrictions on the exponents on a finite union of intervals.

Yu. Lyubarskii described the asymptotics of the sampling constants (the condition number) for lattice families of reproducing kernels in the Fock space when the area of the fundamental domain of the lattice approaches the critical one. Yu. Belov discussed the systems biorthogonal to exact systems of reproducing kernels in Hilbert spaces of analytic functions. In particular, he answered an old question by Nikolski and constructed a model subspace and an exact system of reproducing kernels there with a non-complete biorthogonal system.

The model spaces $K_\Theta$ play an important role in complex analysis, harmonic analysis and Mathematical Physics. The term was coined by N. Nikolskii a long time ago and is suggested by the Szőkefalvi-Nagy–Foiaş model theory of contractions on a Hilbert space. In the simplest (scalar) case, $K_\Theta$ is the orthogonal complement in the Hardy space $H^2$ of the Beurling shift invariant subspaces $\Theta H^2$, where $\Theta$ is an inner function. The central result of Sz. Nagy–Foiaş theory is a theorem describing all contractions of a Hilbert space as compressions to a $K_\Theta$ of a unilateral shift operator. This theorem yields a functional model of a general, abstract linear operator, whence the term of a model space. Model spaces play an important role in approximation theory, too. The first results of these are due to Douglas, Shapiro, Shields and Tumarkin. Also, they are closely related to the phenomenon of pseudo-analytic continuation and to de Branges spaces of entire functions. We note that a de Branges space is isometric to a model subspace $K_B$ where $B$ is a meromorphic Blaschke product. An interesting open question is whether the model space $K_\Theta$ possesses an unconditional basis of reproducing kernels. This is related to a hard problem of J. B. Garnett and P. W. Jones on whether each inner function can be uniformly approximated by interpolating Blaschke products.

Motivated by recent work of Sarason, A. Baranov presented a variety of results concerning the truncated Toeplitz operators $P_\Theta M_\phi$, where $P_\Theta$ is the projector onto the model space $K_\Theta$. 
Another major topic of the meeting was the weighted approximation. J. Brennan discussed the relations between uniform rational approximation and $L^p$-polynomial approximation. H. Hedenmalm proved a uniqueness theorem for the Fourier transforms of measures with support on a hyperbola, related to the Klein–Gordon equation. A. Poltoratski discussed the type and the gap problems in weighted $L^p$ spaces and their relations to the kernels of the Toeplitz operators.

One more topic of interest during the workshop was the corona problem in $\mathbb{B}^n$. B. Wick discussed BMO estimates for this problem using the Koszul complex technique, whereas T. Trent presented his results on the operator version of the corona problem for some multiplier spaces on $\mathbb{B}^n$.

R. Rochberg discussed in his talk geometrical (shape) structures associated with reproducing kernel Hilbert spaces.

N. Arcozzi presented an analog of the Fefferman theorem for the Dirichlet space.

K. Dyakonov presented his results on (local) $abc$ theorems for analytic functions.

R. Zarouf discussed analogs of the Kreiss resolvent condition for matrices with restrictions on the spectrum.

J.-F. Olsen proved an F. and M. Riesz theorem for the Hardy space $H^1(\mathbb{T}^\infty)$.

E. Saksman established the optimal estimate for the growth of the frequently hypercyclic (with respect to the differentiation operator) entire functions. Namely, he proved that for every $c > 0$ there exists an entire frequently hypercyclic function $f$ such that $|f(z)| \leq c|z|^{-1/4}e^{|z|}, |z| > 1$.

E. Abakumov discussed his results on translation cyclic vectors and generating sets in weighted $l^p(\mathbb{Z})$ and $L^p(\mathbb{R})$ spaces.

A. Aleksandrov presented his results on the perturbation (Hölder) smoothness of the functional calculus for the normal operators with respect to the (operator) norm and to the Schatten–von Neumann norm.

A. Nicolau obtained an analog of N. Makarov’s result on the differentiability if the Zygmund class for the case $\mathbb{R}^d, d > 1$. In particular, he proved that every function in the small Zygmund class is differentiable at a set of points of Hausdorff dimension at least 1.

On Wednesday morning a problem session chaired by E. Saksman had been organized. Most of the problems discussed during that session are included at the end of this report. Further open questions were pointed out in many of the talks.

This workshop was organized by Alexander Borichev (Marseille), Raymond Mortini (Metz), Nicolai Nikolski (Bordeaux) and Kristian Seip (Trondheim). Unfortunately, Raymond Mortini, Nicolai Nikolski, and Kristian Seip were unable to participate. All the participants were grateful for the hospitality and the stimulating atmosphere of the Forschungsinstitut Oberwolfach.