Abstract. The workshop brought together experts in finite group theory, \( L^2 \)-cohomology, measured group theory, the theory of lattices in Lie groups, probability and topology. The common object of interest was residually finite groups, that each field investigates from a different angle.


Introduction by the Organisers

The workshop ‘Actions and Invariants of Residually Finite Groups: Asymptotic Methods’ organized by Miklos Abert (Budapest), Damien Gaboriau (Lyon) and Fritz Grunewald (Dusseldorf) was held September 5 - September 11, 2010. Fritz Grunewald tragically and unexpectedly passed away in March 2010. Fritz was a great person and an excellent mathematician and his presence will be missed by all of us. Abert took over as the contact organizer and the MFO has been very efficient with helping him in this new situation.

The workshop aimed to bring together experts in different fields, like finite group theory, \( L^2 \)-cohomology, measured group theory, the theory of lattices in Lie groups, probability and topology. The common object of interest was residually finite groups, that each field investigates from a different angle. There are various group invariants whose asymptotic behaviour on the subgroup lattice of a residually finite group is connected to an interesting analytic invariant of the group. These include the rank and various homological and spectral invariants. The usual setting is to consider a normalized limit of the group invariant on a descending chain in the
The group acts by automorphisms on the corresponding coset tree (a locally finite rooted tree) and this action extends to a measure preserving action on the boundary of the tree. One can connect the dynamics of this boundary action to asymptotic properties of the group.

The meeting was attended by over 50 participants from all over the world and of many fields, including measured group theory, asymptotic group theory, graph theory and topology. There were 24 talks over the five days of the workshop. All talks were 50 minute long except Evija Ribnere’s, which was 30 minutes. This way, less people had the opportunity to talk but they could give a more substantial presentation. This setup seemed to work really well for this particular workshop.

As a general rule, the organizers asked people to talk about specific subjects, not just any nice piece of research. In some cases, this meant sacrificing hearing about some new results from excellent mathematicians that were further away from the workshop’s main directions. What the workshop gained was a strong focus.

The participant body came from a wide range of areas and people typically did not speak each other’s mathematical dialect fluently. To address this, the organizers asked the speakers to put a special emphasis on (and give extra length to) the first, introductory part of their talks. Most of the speakers did this wonderfully. Furthermore, some of the key talks had a major survey aspect, in particular John Wilson gave an introduction to profinite groups, Balazs Szegedy gave an introductory talk on the emerging theory of limits of finite structures, Gabor Elek talked about graph limits in particular and their use in group theory, Narutaka Ozawa gave a survey talk on group measure space von Neumann algebras and measured group theory and Damien Gaboriau gave an introductory talk on orbit equivalence and cost.

On Thursday, some of the talks had a memorial aspect, in particular the talks given by Dan Segal, Martin Bridson and Evija Ribnere put a special emphasis on the mathematics and personality of Fritz Grunewald. Dan Segal kindly agreed to organize a ping-pong tournament in memoriam Fritz, since he liked to play that game in Oberwolfach and was quite good at it. (For the record, Romain Tessera won the tournament with ease.)

There were two talks, where the full proof of the main result could be presented. First, Jan Christoph Schlage-Puchta very recently constructed residually finite, non-amenable torsion $p$-groups, using rank gradient (note that the first, more complicated construction of this kind is due to Denis Osin). The existence of such groups (not necessarily non-amenable, only infinite) was one of the big open questions that brought Golod-Savarevich groups to life. Puchta’s solution is astonishingly simple, in fact it could be presented in a first year undergraduate class without difficulties. Another talk of this kind was from Lukasz Grabowski, a student of Andreas Thom; he has recently found a really transparent way to compute the spectral measure of various group ring elements over the lamplighter group. This topic (versions of the conjectures named after Atiyah) has been considered highly technical and difficult in the past.
Nikolay Nikolov talked about his joint with Miklos Abert on rank gradient of groups, its unexpected connection to cost and how that is related to the rank vs Heegaard genus conjecture.

There were two talks that concentrated on the asymptotic behaviour of invariants for lattices in Lie groups. Tsachik Gelander talked about volume vs. rank of lattices. His main result says that for the family of lattices in a fixed connected semisimple Lie group without compact factors, the rank grows at most linearly in the covolume. Nicolas Bergeron talked about his joint work with Akshay Venkatesh, where they study the growth of torsion in the homology of 3-manifold groups.

Andreas Thom talked about integrality properties of spectral measures. Spectral measures of group ring operators over residually finite groups are weak limits of root distributions of monic polynomials of integer coefficients, and this already means a strong structural restriction on them. It is clear that there must be further restrictions but it is not known exactly what kind.

Emmanuel Breuillard talked about his joint work with Ben Green and Terence Tao about approximate groups and Laszlo Pyber talked about his joint work with Endre Szabo on growth in finite simple groups of Lie type. These two talks were very much related, as the results are highly correlated, but independent – it was very interesting to hear the story from two quite different perspectives.

Mikhail Ershov talked about his joint with Andrei Jaikin-Zapirain on groups of positive weighted deficiency. This talk was related to Jan Christoph Schlage-Puchta’s talk as it generalized some of the notions there. They proved a very exciting result, namely, the existence of ‘residually finite Tarski monsters’, that is, infinite finitely generated residually p-groups where every finitely generated subgroup either has finite index or is finite.

Andrei Jaikin-Zapirain talked about his joint work with Fritz Grunewald, Aline Pinto and Pavel Zalesski on profinite completions and 3-manifold groups.

Roman Sauer talked about his joint work with Peter Limnell and Wolfgang L"uck, where they generalize the L"uck approximation result for arbitrary fields when the fundamental group is amenable. It is an open problem whether such result holds for any group and it is even unclear what exactly to identify the limit with.

Mark V. Sapir talked about his joint work with J. Behrstock and C. Drutu on homomorphisms into the mapping class groups. This work also uses a limit concept, namely asymptotic cones (that uses ultrafilters).

Balint Virag talked about his joint work with Miklos Abert and Yair Glasner. They generalized Kesten’s theorem on vertex transitive graphs to random unimodular networks. This resulted in a new rigidity result on the eigenvalue distributions of finite d-regular graphs.

Mikael Pichot talked about his new work on random groups of intermediate rank. The idea is to generalize the Gromov random group model in a novel way, using a high index finite sheeted cover and then omitting some of the new relations randomly.
Finally, Rostislav Grigorchuk talked about a question of Wiegold and torsion images of Coxeter groups and Todor Tsankov talked about unitary representations of oligomorphic groups. These talks were a bit further from the main directions of the workshop but were certainly interesting to hear.

There were some informal and formal evening sessions with a high attendance. These sessions typically lasted for 2-3 hours and people were free to come in and out anytime. In particular Balazs Szegedy spoke about his new results in higher order Fourier theory, Gabor Elek talked more about graph limits and Miklos Abert talked about profinite actions and weak containment. There was a session run by the two organizers on Friday, that went from 9pm to 2am – it started as a problem session and then evolved into a joyful exchange of ideas and questions. The organizers also attempted to run a young researchers session on one of the evenings.