Abstract. The theory of flows on homogeneous spaces of Lie groups has emerged as a distinct, rapidly advancing subject over the last few decades incorporating ergodic theory, geometry and number theory. The workshop showcased the latest advances in the subject as well as a wide range of applications.

Mathematics Subject Classification (2000): 37D40, 37A25, 22E40, 11N35.

Introduction by the Organisers

The workshop “Flows on homogeneous spaces and arithmetic”, organised by Manfred Einsiedler (ETH Zürich), Dmitry Kleinbock (Brandeis), Elon Lindenstrauss (Hebrew University) and Hee Oh (Brown) was held July 4–10, 2010 and was attended by 52 participants from around the world. The participants ranged from senior leaders in the field to young post-doctoral fellows and PhD students; their range of expertise covered areas from ergodic theory and dynamical systems to automorphic forms, Diophantine approximation and additive combinatorics.

Flows on homogeneous spaces are a class of concrete dynamical systems intimately connected to number theory. Many problems can be approached both via dynamics and via number theoretic and spectral techniques. Recently the interconnection between the dynamics and the arithmetic has flourished, as in many case these two approaches are complementary. The idea of the workshop was to bring together experts in these fields to discuss and collaborate on problems related to homogeneous dynamics, equidistribution, counting integer and rational points, diophantine approximations, and automorphic forms.
The theory of flows on homogeneous spaces received a major impetus in the late 1980’s when Margulis used these dynamical techniques to settle a longstanding conjecture by Oppenheim. Ratner’s well known theorems on rigidity of unipotent flows have found numerous arithmetic applications, often unexpected, e.g. in the study of the number and distribution of integer points in symmetric varieties, values and representations of integer and irrational quadratic forms and even to nonvanishing of certain $L$-functions. A technique developed by Margulis and others to study nondivergence properties of unipotent flows found further applications in metric Diophantine approximation.

Unlike unipotent flows which are well studied, multidimensional diagonal actions are still quite mysterious, though in recent years a substantial progress has been made in their understanding. The talks by Shapira, Tomanov, Wang and Katok dealt with various new phenomena arising from studying such actions, including new examples of irregular orbit closures, while Zamojski talked about applying diagonal actions to a certain counting problem.

One of the central recent events in the area has been a breakthrough work of Benoist and Quint on classification of invariant and stationary measures for actions of Zariski dense subgroups of Lie groups, not necessarily generated by unipotent elements. Quint gave a series of two talks on this subject, and in addition Eskin spoke about a new development in classification of invariant measures for some actions on the moduli space of Riemann surfaces utilizing some ideas due to Benoist and Quint.

Another area which has attracted a lot of attention during the workshop was equidistribution of horocycles and their geodesic translates on homogeneous spaces of infinite volume. Talks of Oh, Shah, Marklof, Roblin, Schapira, Paulin discussed such results, and in many cases – applications to counting problems. The theme of counting and equidistribution can be also approached by methods coming from the theory of automorphic forms, or by studying the spectral gap of certain groups. This has been highlighted in talks by Fuchs, Kontorovich, Kowalski, Ghosh, Gorodnik, while Breuillard and Varju discussed families of expanders arising from certain groups. Green’s talk on his joint work with Tao centered on a connection between problems in additive combinatorics and equidistribution of nilflows. Another approach to counting problems, based on integral inequalities on the space of lattices, was surveyed by Margulis.

The workshop went very well; in order to leave enough time for fruitful discussions, the number of talks (50 minutes long) was limited to five per day, and to 23 altogether. On Thursday evening we had a session for short communications (5 minute long talks and five minute intervals for discussion), which allowed many young participants to introduce themselves and the circle of problems they have been working on. The traditional Wednesday afternoon hike has successfully contributed to cheerful and productive atmosphere of the workshop!