Abstract. Algebraic geometry studies properties of specific algebraic varieties, on the one hand, and moduli spaces of all varieties of fixed topological type on the other hand. Of special importance is the moduli space of curves, whose properties are subject of ongoing research. The rationality versus general type question of these and related spaces is of classical and also very modern interest with recent progress presented in the conference. Certain different birational models of the moduli space of curves and maps have an interpretation as moduli spaces of singular curves and maps. For specific varieties a wide range of questions was addressed, including extrinsic questions (syzygies, the k-secant lemma) and intrinsic ones (generalization of notions of positivity of line bundles, closure operations on ideals and sheaves).

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Introduction by the Organisers

The workshop Classical Algebraic Geometry held from June 20th to June 26th 2010 at the "Mathematisches Forschungsinstitut Oberwolfach" was organized by David Eisenbud (Berkeley), Frank-Olaf Schreyer (Saarbrücken), Ravi Vakil (Stanford) and Claire Voisin (Paris). It was very well attended with over 50 participants from the United States, Canada, United Kingdom, Italy, France, Poland and Germany. In total there were 18 one hour talks with a maximum of four talks a day and a session with short presentations allowing young participants to give 10 minute outlines of their current work. This schedule left plenty of room for many informal discussions and work in smaller groups.
The extended abstracts give a detailed account of the broad variety of topics of the meeting, many of them classical questions in algebraic geometry discussed with modern methods. Although it would be nice if we could mention all talks here, we should focus on a couple of highlights:

- The conference opened with Gavril Farkas’ lecture on his major new work on Green’s conjecture on syzygies of canonical curves. The previous major breakthrough was Voisin’s proof earlier this decade that Green’s conjecture holds for smooth curves lying on a K3 surface with Picard group of rank 1, and Aprodu’s result (building on Voisin) that Green’s conjecture holds for all curves satisfying the linear growth condition, thus turning Green’s conjecture into a question in Brill-Noether theory. Farkas explained his recent proof that Green’s conjecture in fact holds for every smooth curve lying on an arbitrary K3 surface.

- Daniel Erman, who received his Ph.D. shortly before arriving, reported on joint work with Melanie Wood (another recent Ph.D., and an AIM five-year-fellow). Their work deals with the space of degree $n$ covers of varieties (or schemes), which they elegantly interpret in terms of the space (stack) of moduli of points. If $n \leq 5$, beautiful descriptions involving matrix presentations (or geometric variations thereof) lead to explicit useful results. (One prominent example is the celebrated work in number theory of Manjul Bhargava.) They begin to address the case $n = 6$, making clear what makes this case difficult. A special case are configurations arising from the Gulliksen-Negard complex, and they give nontrivial restrictions on the class of sextic covers which can arise from such a construction.

- The recent development of tropical geometry has had a significant impact on algebraic geometry, by turning many classical problems into piecewise linear problems that one can work with, either to calculate, or to prove theorems. Sam Payne reported on recent work with Brian Osserman developing intersection theory in tropical geometry, in such a way that the results will translate into classical algebraic geometry. This requires the systematic development of dimension theory and intersection theory on decidedly non-classical objects: schemes of finite type over non-Noetherian valuation rings of rank 1. Their general theory promises to subsume and extend earlier ad hoc methods, and will substantially change the subject.

- The concluding lecture of Joe Harris tied together a number of recent advances on the birational geometry of parameter spaces of curves. Different compactifications of the space of curves (both by themselves, and in projective space) have proved useful in a number of contexts. It has been recently observed (first by Hassett and Keel) that these different compactifications are often related in geometrically meaningful ways. For example, D. Chen, Coskun, and Crissman have shown that many of the spaces of rational curves in projective space arise in this way. Also, Viscardi has generalized recent work of Smyth and others on genus 1 curves to find a small compactification of genus 1 curves in projective space, which one
might hope is related to the recent “stable pairs” construction of Marian, Oprea, and Pandharipande.

The young participants’ presentations by Michele Bolognesi, Dawei Chen, Christian Christensen, Thomas Dedieu, Florian Geiss, Andreas Höring, Grzegorz Kapustka, Paul Larsen and Margherita Lelli-Chiesa also covered a widespread variety of topics from moduli of curves, vector bundles, K3 surfaces to abelian varieties.