Geometry, Quantum Fields, and Strings: Categorial Aspects

Organised by
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Abstract. Currently, in the interaction between string theory, quantum field theory and topology, there is an increased use of category-theoretic methods. Independent developments (e.g. the categorification of knot invariants, bundle gerbes and topological field theories on extended cobordism categories) have put higher categories in the focus.

The workshop has brought together researchers working on diverse problems in which categorical ideas play a significant role.

Mathematics Subject Classification (2000): 81T, in particular 81T45 and 81T13.

Introduction by the Organisers

The workshop Geometry, Quantum Fields, and Strings: Categorial Aspects, organised by Peter Bouwknegt (Australian National University, Canberra), Dan Freed (University of Texas, Austin), and Christoph Schweigert (University of Hamburg) was held June 6th–June 12th, 2010. The meeting was attended by 52 participants from all continents.

18 talks of one hour each were contributed to the workshop. Moreover, young researchers were offered the possibility to present short contributions. On Monday and Wednesday evening a total of 11 short talks were delivered. We would like to stress the high quality and level of interest of these contributions. The two sessions have received much attention and have led to much additional scientific discussion about the work of younger participants. For this reason, these contributions are covered in these proceedings as well.

Another special event was a panel discussion on Tuesday evening on the topic “Whither the interaction of Geometry-QFT-String?”. Chaired by Dan Freed and...
under the lively participation of the audience, Kevin Costello, Michael Douglas, Greg Moore and Tony Pantev exchanged their point of view on recent and present interactions between mathematics and physics in the area of quantum field theory. There was a broad agreement that the field is in rapid progress and presents many exciting challenges that necessitate the interaction of researchers of different background. Homotopical techniques and generalized cohomology theories can be expected to play an increasingly important role in the study of quantum field theories and string theories.

The much of the work presented during this workshop could be described as “mathematics inspired by string theory and quantum field theory”. Most of the contributions to the workshop were related to the following three main topics that are strongly interrelated:

1. (Higher) categorial descriptions for quantum field theories, in particular for topological quantum field theories and extended versions of quantum field theories.
2. Structures related to moduli spaces.
3. Higher categorial structures of string backgrounds.

We summarize the contributions to this workshop according to these three subfields.

A construction of topological field theories based on Fukaya categories has been explained by Chris Woodward. A three-category of chiral conformal field theories has been discussed in an operator algebraic approach by Arthur Bartels; Alexei Davydov explained how aspects of the classification of rational chiral conformal field theories can be captured in the definition of a Witt group of modular categories. In Michael Douglas’ talk, the space of quantum field theories, in particular two-dimensional conformal field theories, has been addressed from a completely different point of view; in the form of defects, higher categorial structures have been central to this approach as well.

Generalizations of Knizhnik-Zamolodchikov equations for conformal blocks have been presented in the talk of Valerio Toledano Laredo. Constantin Teleman discussed some aspects of (extended) topological field theories in two dimensions related to Gromov-Witten invariants. Kevin Costello explained his notion of factorization algebra, an adaptation of vertex algebras to a smooth setting, and its applications to the Witten genus. Inspired by older work on anomalies in quantum field theories in a Hamiltonian framework, Jouko Mickelsson proposed in particular applications of gerbal representations to quantum field theory.

In the young researchers’ session, the contributions of Orit Davidovich on extended topological field theories from state sum models and by Konrad Waldorf on gauge anomalies in two-dimensional bosonic sigma models complemented this circle of topics.

Dennis Gaitsgory’s and Craig Westerland’s talk discussed different aspects of moduli spaces of bundles. Structured moduli spaces of curves were discussed in Ezra Getzler’s contribution in a symplectic setting and Nathalie Wahl presented
a graphical calculus to the Hochschild homology of structured algebras motivated by string topology.

The third topic about string backgrounds included in particular a careful discussion of orientifold backgrounds by Greg Moore in terms of differentially refined and twisted cohomology theories. Such theories are also important in the discussion of T-dualities: a perspective on T-duality using a Lagrangian formalism for sigma-models was given in Kenta Hori’s talk; Mathai Varghese discussed T-duality in the presence of background fluxes and explained the need to include non-commutative geometry and non-trivial associators in the picture. In the young researchers’ session, this was complemented by contributions by Alexander Kahle (touching also aspects of differential refinements of cohomology theories) and Rishni Ratnam about non-commutative torus bundles.

Twisted K-theory and its relation to the Verlinde algebra was one topic of Igor Kriz’ talk. Yan Soibelman presented an algebraic approach to motivic Donaldson-Thomas invariants based on Calabi-Yau categories. Ludmil Katzarkov finally discussed non-abelian mixed Hodge structures.

The contributions of Igor Bakovic on 2-stacks, of Dan Berwick-Evans on supersymmetric sigma-models, of Braxton Collier on categorial Lie algebras, of Thomas Nikolaus on algebraic methods for higher categories and Hisham Sati on the geometry of membranes in the young researchers’ session added important complements to these topics.