

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mini-Workshop: The Escaping Set in Transcendental Dynamics

Organised by
Walter Bergweiler (Kiel)
Gwyneth Stallard (Milton Keynes)

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ABSTRACT. The escaping set of a transcendental entire or meromorphic function consists of all points which tend to infinity under iteration. Its importance in transcendental dynamics has increased significantly in recent years. The workshop focussed on a study of this set. The topics considered include the geometry of the escaping set, its Hausdorff dimension, its relation to the Julia set, and various subsets of the escaping set defined in terms of escape rates.

Mathematics Subject Classification (2000): 37F10, 30D05.

Introduction by the Organisers

Transcendental dynamics studies the behavior of transcendental entire functions under iteration. The main object considered has been the *Julia set* $J(f)$ of a function f which is defined as the set where the iterates f^n of f do not form a normal family. In recent years, however, it has become apparent that the *escaping set*

$$I(f) = \left\{ z \in \mathbb{C} : \lim_{n \rightarrow \infty} f^n(z) = \infty \right\}$$

also plays a key role in transcendental dynamics. There is a close connection between the two sets since we always have $J(f) = \partial I(f)$.

The workshop focussed on the escaping set and its role in complex dynamics. It was attended by 16 participants who gave talks on different aspects of the subject. In addition, there was a problem session where open problems were presented (and discussed). The unique atmosphere of Oberwolfach also stimulated many mathematical discussions among the participants, and sufficient time for this was left as well.

In the first talk of the conference, L. Rempe reported on the most recent developments in connection with a question of Eremenko who asked whether every component of $I(f)$ is unbounded for transcendental entire f . While there has been considerable progress recently, the problem is still open. The talks by X. Jarque, J. Taixés and H. Mihaljević-Brandt also addressed questions concerning the topology of the escaping set and the Julia set.

In complex dynamics it is of particular interest to consider specific families of functions such as the exponential family λe^z and to study how the dynamics depend on the parameter. This was addressed in the talks by X. Jarque, A. Badenska, K. Barański, D. Schleicher and J. Peter.

Another topic of major importance in complex dynamics is the Hausdorff dimension of the escaping set, the Julia set, and related sets. These topics were addressed in the talks by A. Zdunik, B. Karpińska, M. Urbanski and J. Peter.

For quasiregular maps in higher dimensions, there is no obvious definition of the Julia set. The escaping set can still be defined, however. A. Eremenko and D. Nicks reported on recent results which show that some results about the escaping set can in fact be carried over from the plane to higher dimensions.

Components of the Fatou set which lie in the escaping set, as well as different escapes rates of points in the escaping set, were considered in the talks by W. Bergweiler, P. J. Rippon and G. M. Stallard.

Overall this was a very successful meeting with excellent talks, lively discussions and a fruitful exchange of ideas.