Abstract. Transfer operators and their spectral theory provide a unifying framework for studying stochastic properties of chaotic deterministic dynamical systems. The goal of this workshop was to widen the class of systems that can be analysed in this way and to discuss and present new applications.

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Introduction by the Organisers

Transfer operators are linear operators associated to discrete- or continuous-time dynamical systems with some hyperbolicity. Acting on suitable Banach spaces, they often have Perron-Frobenius type spectrum, although it can be quite tricky to find a proper Banach space on which this can be proved and be exploited to obtain statistical information about the dynamical system (SRB measures and other Gibbs states, exponential decay of correlations, statistical stability, probabilistic limit theorems, linear response, . . . ). The first results date back to the 1960’s (D. Ruelle), and the three organisers have contributed to this theory since the 1980’s (see [1] for a detailed discussion and results of such a theory). In the last decade a fresh point of view has gradually emerged, where most of the combinatorial (and sometimes quite artificial) constructions used previously, like Markov partitions, Young towers, cluster expansions etc., can be avoided by introducing suitable Banach spaces. This not only allows to give simpler proofs of known results on smooth hyperbolic systems, but also to extend the theory to piecewise smooth systems,
physically hyperbolic systems, or systems in infinite dimensions (such as coupled map lattices). What is common to most of these situations is that rather complicated arguments of combinatorial flavour (book-keeping of iterated singularities, spatial dependencies etc.) are captured by norm estimates in judiciously chosen Banach spaces. These techniques not only apply to dynamical systems but also to many Markov chains including various time series models of current interest (see [2]). All participants of the workshop are actively working on the quest for suitable Banach spaces or on exploiting the spectral information thus obtained for a deeper understanding of the dynamical systems under study (or on both).

In order to leave enough time for discussions and cooperations, the number of talks was limited to twelve. They all were followed by long and lively discussions, and some of them were continued the following day by demand of the audience.

For the development of new Banach spaces the dialogue between tools developed in semiclassical analysis (related to quantum mechanics) and the techniques introduced recently for transfer operators of chaotic dynamical systems was most fruitful. A number of new ideas originated from this which hopefully will produce interesting results in the near future. But also the detailed comparison of the special advantages and difficulties of the approaches based on the one hand on Triebel-type Banach spaces and on the other hand on more geometrically defined spaces lead to new insights into the particular problems associated with the spectral approach to piecewise hyperbolic systems. At the end of the workshop the road to an operator treatment of the Sinai-billiard flow was visible.

Among the applications of transfer operator theory were a strikingly new and very general approach to prove almost sure invariance principles for partial sum processes generated by chaotic dynamical systems, precise results on linear response theory for Benedicks-Carleson unimodal maps, applications to Poincaré sums and sum-level sets of continued fraction expansions, genericity results on the Ruelle spectrum of transfer operators, and limit theorems for the statistics of systems with shrinking targets.

Talks on complex cone contractions and on self-consistent (nonlinear) Perron-Frobenius operators indicated related directions of research.

In summary it is clear that a new unifying point of view in the study of hyperbolic dynamical systems is emerging. Ideally such a new approach will encompass both discrete and continuous time, both smooth and piecewise smooth systems, both conservative and dissipative phenomena, both finite and infinite dimensional models. We believe that this workshop, by bringing together different approaches to and people working on different, but related, facets of such project, has considerably advanced the above research program and that its influence will be felt in the future development of the field.

References
