Mini-Workshop: Product Systems and Independence in Quantum Dynamics

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Abstract. Quantum dynamics, both reversible (i.e., closed quantum systems) and irreversible (i.e., open quantum systems), gives rise to product systems of Hilbert spaces or, more generally, of Hilbert modules. When we consider reversible dynamics that dilates an irreversible dynamics, then the product system of the latter is equal to the product system of the former (or is contained in a unique way). Whenever the dynamics is on a proper subalgebra of the algebra of all bounded operators on a Hilbert space, in particular, when the open system is classical (commutative) it is indispensable that we use Hilbert modules.

The product system of a reversible dynamics is intimately related to a filtration of subalgebras that are independent in a state or conditionally independent in a conditional expectation of the reversible system. This has been illustrated in many concrete dilations that have been obtained with the help of quantum stochastic calculus. Here the underlying Fock space or module determines the sort of quantum independence underlying the reversible system.

The mini-workshop brought together experts from quantum dynamics, product systems and quantum independence who have contributed to the general theory or who have studied intriguing examples. As the implications of the tight relationship between product systems and independence had so far been largely neglected, we expect from our mini-workshop a strong innovative impulse to this field.

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Introduction by the Organisers

The analysis of dynamical systems, classical or quantum, reversible or irreversible, is a central issue when applying mathematics to natural sciences, economy, sociology, computer and information sciences, and others.

Since William Arveson’s fundamental work on product systems of Hilbert spaces (Arveson systems) we know that product systems play an outstanding role in the classification of quantum dynamics. Boris Tsirelson’s large classes of examples of spatial product systems of Arveson systems showed that product systems are intimately related to stochastic independence. If classical dynamics should be included, then Hilbert modules are an indispensable tool.

The Mini-Workshop Product Systems and Independence in Quantum Dynamics, concentrated on these connections, with the goal to open a new promising line of research. To achieve this goal we brought together leading experts in the relevant fields. In the first two days, the state of the art was presented in a series of lectures on selected important topics. During last three days more specific aspects were developed in one-hour research talks. The files of some lectures and talks can be found on the web page

http://www.math-inf.uni-greifswald.de/algebra/q-dyn/

of the workshop.

Particularly exciting were the open problems sessions. Trying to open a new line of research, it appeared natural to us scheduling two open problem sessions. As a matter of surprise to us, each open problem session took almost two hours. We think it is not possible to underline better the resonance our concept had. It is our wish to share these open problems, and a detailed description can be found in the Open Problems Section.

We wish to thank all participants, whose participation made the workshop a success. We would also like to express our deep gratitude to the MFO, its Director Prof. Dr. Greuel, the Gesellschaft für Mathematische Forschung e.V., and last but surely not least, the staff of the MFO, for offering to us the opportunity to organize this Mini-Workshop.

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