Introduction by the Organisers

1. General background

In the last thirty years quite some initiatives evolved and much material was developed for using the history of mathematics in the teaching of mathematics at all levels. There is a growing consensus that historical work of pupils and students may contribute to further through:

- providing insights into the development of mathematical concepts;
- developing a deeper understanding of the role of mathematics in our surrounding world and its relation to applications, culture and philosophy; and
- encouraging the perception of the subjective dimensions of mathematics: of aims and intentions in the building of mathematical concepts and algorithms, of alternative methods and of personal and creative aspects.

Among the various possible activities by which historical aspects might be integrated into the teaching of mathematics, the study of an original source is the most demanding and the most time consuming. It requires a detailed and deep understanding of the mathematics in question, of the time when it was written and of the general context of ideas. The aspect of language becomes important in ways which are completely new compared with usual practices of mathematics teaching.
Thus, reading a source is an especially ambitious enterprise, but rewarding and capable of substantially deepening the mathematical understanding.

In principle, the aims and effects which might be pursued through the study of an original source will not be different from those attained by other types of historical activities. However, there are some ideas which are specifically supported by reading mathematical sources.

(1) Studying an original source replaces the usual with something different: it allows student and teacher to see mathematics as an intellectual activity, rather than as just a corpus of knowledge or a set of techniques. For example, Newton’s letter to Leibniz of 1676 in which he described how as a young man of 22 years he arrived at the general binomial formula (a cornerstone in his fluxional calculus) is a unique document for a process of mathematical invention progressing by bold generalisations and analogies. Through the reading of the letter, the student more or less feels the presence of the inventor.

(2) Integrating sources in mathematics challenges the learner’s perceptions through making the familiar unfamiliar. Coming to grips with a historical text can cause a reorientation of the learner’s views and thus deepen his or her mathematical understanding. All too often in teaching, concepts appear as if already existing and they are manipulated with no thought for their construction. Sources remind students that these concepts were invented and that such invention did not happen all by itself. As an example, we might refer to Leibniz’ version of the calculus. There are many experiences which show that students are motivated to reflect about the limit approach to calculus when they study Leibniz’ way of dealing with infinitely small quantities. Also the teacher may gain insight by concentrating on the unfamiliar. It is often difficult enough to cope with unexpected solutions by students; however, studying sources enables to the teacher and students to keep an open mind.

(3) Integrating the study of sources in mathematics education invites the learner to place the development of mathematics in the scientific and technological context of a particular time and in the history of ideas and societies. One of many examples from antiquity to the present is provided by Heron’s textbook (1st century A.D.) on land surveying called The Dioptra. Reading parts of it connects the topic of similarity to the context of ancient surveying techniques and shows the astonishingly high achievements of ancient engineers in this and other areas. Such sources may as well provoke students to engage in practical activities (simulations, measurements, theatre), which otherwise would not come to their mind or to the mind of their teacher.

(4) Reading a source is a type of activity which is oriented more to processes of understanding than to final results. The complete meaning of a historical text may remain open, and it occurs quite often that the same text leads to different readings. Of course, this does not entail arbitrariness. The
reader has to give reasons in support of his or her interpretation. As an example we refer to the highly interesting story of negative and/or complex numbers. Reading sources about this topic poses in every case the question whether, and if yes, in which sense these creations were understood as legitimate numbers in different historical times. Doubts that students themselves have from time to time are reflected by the doubts that existed through the ages.

(5) When working with original sources at least three different languages interact in the classroom: the language of the source, the modern terminology of the mathematical topic in question and the everyday language which has evolved in the classroom. This requires of the learner competencies of translation and switching between these languages which are highly desirable from an educational point of view since communication between expert mathematicians and people who want a problem solved mathematically is one of the main problems of applying mathematics.

2. Theoretical and practical orientations

The mini-workshop comprised sessions of different types. Most of the meetings were devoted to traditional presentations of papers. On the other hand, in some sessions the participants discussed the needs and aims of the future development of the field. As a result, research questions were identified which evolved from work in the past and might be helpful in orienting future work. They reflect central issues related to the integration of original sources from the history of mathematics into mathematics education. Each of the questions addresses both the learning of mathematics (by secondary school and university students and by prospective or in-service teachers) and the teaching of mathematics (at the secondary and university levels). In both cases, each of the questions retains its general formulation; however, each is approached differently by the authors according to the target population and their intended educational goals. Thus each question may have more than one answer.

(1) What are the possible epistemological/theoretical basis and frameworks for research and development towards the integration of original sources into the teaching and learning of mathematics?

(2) What are the characteristics of viable models for implementing the integration of original sources in the teaching and learning of mathematics?

(3) What is the actual impact of these models on students’ and teachers’ learning and understanding of mathematics, and on teachers’ teaching practices?

(4) How can historical research and practice inspire, impact, support or supply explanatory frameworks and working tools for research on learning and teaching mathematics?

(5) How can research and practice in mathematics education inspire, support and broaden the research in the history of mathematics in general, and on original sources in particular?
Another issue that came up in the workshop was the problem of upscaling. There is no reason to believe that teaching which is done by an enthusiast with good results can easily and successfully be repeated by the average teacher. It is a welcome development that new materials are being published and that research is being done on projects where “average teachers” are doing the teaching.

In many countries mathematics education standards are in the process of being elaborated. These standards often appear as collections of mathematical problems. The approach of reading sources can be successful in the future only if the community will produce problems in a format adequate to be included among these standard problems. Some contributions during the meeting showed that this is in fact possible.

A related issue is the important role history of mathematics, particularly the reading of sources, might play in the training of teachers for all levels. Studying sources can provide awareness for subtleties in the meaning of mathematical concepts which cannot be afforded otherwise. Thus, the sensitivity of teachers in regard to content-related difficulties of their students might be considerably enhanced. The workshop showed that many activities are in place worldwide which try to take advantage of an approach which includes the study of sources.

3. The Workshop

Most of the contributions during the workshop were related to and inspired by one or more of the research questions outlined in the previous section.

- An important development in recent years is that more empirical research studies on the integration of original sources are being done, many of which include a large number of students. A few of them were presented here (Glaubitz; Clark; Peters; van Maanen, reporting about his student Iris van Gulik-Gulikers).
- Other talks were focusing on theoretical issues based on examples from practice (Arcavi; Bardini, Radford).
- Some papers gave examples from the presenters’ own practice with comments on their theoretical background (Barbin; Dematté; Wann-Sheng Horng; van Maanen; Rasfeld; Reich).
- In two sessions the audience was invited to take part in working on historical sources (Fengeleley; Jahnke).
- One talk gave an overview on curricula, textbooks and teachers and their roles in making history of mathematics part of mathematics education (Smestad).

The organizers thank the Institute staff for providing a comfortable environment to the participants.