Heterotic Strings, Derived Categories, and Stacks

Organised by
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Abstract. This workshop brought together both mathematicians and physicists interested in mathematical aspects of heterotic strings and physical aspects of derived categories and stacks. These three topics in mathematics and physics are all involved in modern approaches to and extensions of mirror symmetry, and much of the technical machinery in understanding their physics and mathematics overlap, so by bringing together experts in these areas we hope to help spur further developments.

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Introduction by the Organisers

The miniworkshop *Heterotic strings, derived categories, and stacks*, organised by Björn Andreas (Berlin), Emanuel Scheidegger (Vienna) and Eric Sharpe (Utah) was held November 13th–November 19th, 2005. This meeting was well attended with 14 participants with broad geographic representation. This workshop was a nice blend of researchers with various backgrounds in both mathematics and physics.

The three topics represent areas of mathematics and physics with significant technical overlap. Heterotic strings are types of string theories whose compactifications involve complex Kähler manifolds with holomorphic vector bundles, and most of the complications revolve around those vector bundles. Derived categories (of coherent sheaves) have an obvious mathematical link with holomorphic vector bundles, and appear physically in studies of D-brane/antibrane systems. Details of the physical model in which derived categories enter physics are also closely related to the details of the physical model in which stacks enter physics: in each case, only a distinguished subclass of presentations can be realized physically, and
the nonuniqueness of presentations in that subclass is conjectured to be washed out by a physical process called renormalization group flow.

These topics also form elements of generalizations of a conjectured generalization of "mirror symmetry." Mirror symmetry is a symmetry exchanging pairs of complex Kähler manifolds with trivial canonical bundle. It has been of interest to algebraic geometers because it provides a new approach to enumerative geometry: (usually difficult) curve-counting questions were mapped to comparatively trivial questions about the mirror manifold. Mirror symmetry was originally developed for spaces, but recently has been extended to stacks. One of the conjectured generalizations of mirror symmetry, known as "(0,2) mirror symmetry," exchanges pairs consisting of complex Kähler manifolds with holomorphic vector bundles, and is an analogue of ordinary mirror symmetry for heterotic strings. Another generalization, known as "homological mirror symmetry," exchanges derived categories of coherent sheaves on one of the mirrors with a derived Fukaya category of the other. As the topics of this miniworkshop show up in these new areas of mirror symmetry, this miniworkshop could have instead been titled "New developments in mirror symmetry."

Since understanding these topics involves an interplay between mathematics and physics, for this miniworkshop we brought together a collection of both mathematicians and physicists.

B. Andreas, V. Braun, and E. Scheidegger spoke specifically on mathematical aspects of heterotic strings, and E. Sharpe gave an overview of a few current problems in heterotic strings. A. Tomasiello spoke on mirror symmetry in flux backgrounds, using ideas recently developed by Hitchin to extend mirror symmetry for type II strings. (The same ideas can also, it is thought, be used to solve certain technical problems in understanding heterotic strings in flux backgrounds, as discussed in E. Sharpe’s talk.) D. Ploog spoke on general aspects of derived categories and Fourier-Mukai transforms, then U. Bruzzo and D. Hernandez Ruiperez gave a collection of talks on Fourier-Mukai transforms, relevant to both derived categories (encoding automorphisms thereof) and heterotic strings (encoding T-dualities). E. Macri spoke on pi-stability, a physical aspect of derived categories. K.-G. Schlesinger and C. Lazaroiu spoke on $A_\infty$ and $L_\infty$ algebras, as relevant to open and closed string field theory, and which play a role in the physical understanding of derived categories. Finally, E. Sharpe and P. Horja gave a collection of talks on physical aspects of stacks.