Abstract. The workshop studies Hamilton-Ricci flow of Riemannian metrics on 3-manifolds. The participants give detailed technical lectures on recent work of G. Perelman concerning a priori estimates and surgeries during the flow. The workshop was able to verify major sections of Perelman’s work and identified points that need a more detailed exposition.

Mathematics Subject Classification (2000): 53C44.

Introduction by the Organisers

The motion of a Riemannian metric along its Ricci curvature,

\[ \frac{d}{dt} g_{ij} = -2R_{ij}(g(t)) \]

was proposed in 1982 by Richard S. Hamilton as a geometric version of the heat equation suitable for uniformizing and smoothing the geometry of a given initial Riemannian manifold \((M^3, g_0)\). Hamilton’s work has opened up the whole area of geometric evolution equations, leading to the discovery of new phenomena in these equations and to topological applications such as the classification of 3-manifolds of positive Ricci-curvature and certain 4-manifolds. Recent work of G. Perelman has indicated how to approach a proof of the Poincaré-conjecture and the Thurston Geometrization conjecture for 3-manifolds using Hamilton-Ricci flow.

The mini workshop has concentrated on a thorough technical investigation of that part of the work of Perelman that is related to Hamilton-Ricci flow with surgeries on a finite time interval. Together with further work of Perelman and also Colding-Minicozzi this part of Perelman’s work implies a proof of the Poincaré
The efforts of the workshop were greatly helped by previous work of other mathematicians on Perelman’s work, e.g. the notes of B. Kleiner and J. Lott.

The workshop was able to confirm major sections in the two papers of G. Perelman including the entropy- and reduced volume estimates, the compactness properties of ancient solutions to the flow and the surgery construction. It was also able to reinterpret several arguments involving Alexandrov spaces from the viewpoint of smooth Differential Geometry.

When the workshop had to come to an end, its participants agreed that it would be very desirable to establish self-contained expositions of the following points:

1. The boundedness of the curvature \( \sup_{B_r(x)} R \leq c(\rho) \) in the proof of the approximation theorem 1.12.1 of Perelman’s first paper.
3. The survival of the reduced volume estimate past surgeries.
4. The uniform control of a fixed scale \( \rho > 0 \) past all surgeries on a finite time interval, below which the approximation theorem applies.

It seems that detailed self-contained expositions of (3) and in particular (4) require more effort than (1) and (2).